Math 130A Practice Final, Winter 2003, Lindblad.

The problems below are some standard types of problems that might be on the final. The final is cumulative and a large part of the problems will be about linear systems, similar to those on the midterm. A good review for these types of problems is to go over the midterm and practice midterm again. We only give some problems from the later sections. Most of the problems below are taken from the book and they are either solved examples in the book or homeworks that was assigned.

1. (Problem 7.2.1c) Let $A \in L(\mathbf{R}^3)$ be a 3×3 matrix with eigenvalues $\lambda < 0$, $\mu = a + bi$, $a > 0$ and $b > 0$. Note that e^{At} is a hyperbolic flow and sketch the phase portrait. Is the origin a sink, a source, a saddle or neither? Is the origin stable, asymptotically stable or unstable?

2. (Example 9.3.1) Consider the dynamical system

$$x' = 2y(z-1), \quad y' = -x(z-1), \quad z' = -z^3$$

Find the equilibrium points and determine if they are stable, asymptotically stable or unstable. Hint: Try to find a Liapunov function of the form $V = ax^2 + by^2 + cz^2$.

3. (Problem 9.4.1f) Let $V(x, y) = x^2(x-1) + y^2(y-2) + z^2$. Consider the system

$$(x', y') = -\operatorname{grad} V(x, y)$$

Find the equilibria and classify them as to stability, asymptotic stability and instability. Sketch the phase portrait of the trajectories and the level sets of V in the same diagram.

4. (Section 10.2 and Problem 10.2.1) Consider the system

$$x' = y - f(x), \qquad y' = -x$$

Find the equilibrium points and determine if they are a source or a sink, depending on f. Are the equilibrium points asymptotically stable, stable or unstable. Sketch the phase portrait if $f(x) = x^2$.

5. (Problem 10.4.1) Find all values of μ which are bifurcation points for

$$x' = \mu x + y, \qquad y' = x - 2y$$