

**Lecture 14: Three dimensional linear systems.**

**Recall: Normal forms for  $3 \times 3$  matrices:**

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \nu \end{pmatrix}, \quad \lambda, \mu, \nu \text{ real.}$$

$$\begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \mu \end{pmatrix}, \quad \lambda, \mu \text{ real.}$$

$$\begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}, \quad \lambda \text{ real.}$$

$$\begin{pmatrix} \alpha & \beta & 0 \\ -\beta & \alpha & 0 \\ 0 & 0 & \lambda \end{pmatrix}, \quad \alpha, \beta, \lambda \text{ real.}$$

**Normal forms for  $4 \times 4$  matrices:**

$$\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \quad \text{where } A, B \text{ are } 2 \times 2 \text{ matrices in canonical form.}$$

$$\begin{pmatrix} A & I \\ 0 & A \end{pmatrix} \quad \text{where } A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \mu \end{pmatrix} \quad \lambda, \mu \text{ real.}$$

$$\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix} \quad \lambda \text{ real.}$$

We covered the examples on pages 97 and 98.