

Problem 1 (50%). Let $A = \begin{pmatrix} 0 & -2 \\ 1 & -3 \end{pmatrix}$.

- (a) Calculate a matrix T and a diagonal matrix D so that $D = T^{-1}AT$.
- (b) Write down the general solution of the equation $Y' = DY$ and sketch the phase portrait.
- (c) Write down the general solution of the equation $X' = AX$.
- (d) For which initial values $X(0)$ does the solution $X(t)$ to the equation $X' = AX$ travel in a straight line?

Problem 2 (50%). (a) Write down the general solution of the harmonic oscillator $x'' - 2x' + 2x = 0$.

- (b) Describe the behavior of $x(t)$ as $t \rightarrow \infty$.