

MATH 140A. HOMEWORK 2. DUE WEDNESDAY, JANUARY 21, IN SECTION
Chapter 2, # 2, 3, 4.

H1. True or false? For all sets A, B and functions $f : A \rightarrow B$ and subsets $C \subset A$ and $D \subset B$,

$$\begin{array}{ll} (a) f(f^{-1})(D) = D, & (b) f^{-1}(f(C)) = C, \\ (c) f^{-1}(f(f^{-1}(D))) = f^{-1}(D), & (d) f(f^{-1}(f(C))) = f(C). \end{array}$$

H2. Prove carefully that

$$(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B.$$

Suppose a_1, a_2, \dots are integers with $0 \leq a_k \leq 9$, and if a_0 is an integer. We define

$$a_0.a_1a_2\dots a_n := a_0 + \frac{a_1}{10} + \frac{a_2}{100} + \dots + \frac{a_n}{10^n}.$$

H3. Describe the rational numbers p/q which are equal to finite decimal expansions of the form $a_0.a_1a_2\dots a_n$.

H4. Find the rational number whose decimal expansion is $0.334444\dots$

H3. Suppose a_1, a_2, \dots are integers with $0 \leq a_k \leq 9$, and if a_0 is an integer. We say that $a_0.a_1a_2a_3\dots$ is an infinite decimal expansion of x , if

$$x = \sup\{a_0.a_1a_2\dots a_n : n = 1, 2, \dots\}.$$

Determine which positive real numbers x have more than one decimal expansion.

H5. Suppose x is a positive real number. Let a_0 be the largest integer less than or equal to x and for $n = 1, 2, \dots$, define integers a_1, a_2, \dots recursively so that a_n is the largest integer with $0 \leq a_n \leq 9$ and

$$a_0.a_1a_2\dots a_n \leq x.$$

Let

$$A = \{a_0.a_1a_2\dots a_n : n = 1, 2, \dots\}.$$

Show that $x = \sup A$.