

MATH 140A. HOMEWORK 6. DUE WEDNESDAY, FEBRUARY 18

Chapter 2 problems 27, 28, 29, 30.

(**Chapter 3** problems 1, 2, 3. These problems are postponed to next week due to popular request.)

H0. Suppose $r > 0$. Show that there exists $n > 0$ such that $1/2^n < r$.

H1. Suppose that for $n = 1, 2, 3, \dots$ the set I_n is a k -cell and $I_{n+1} \subset I_n$. Suppose in addition that

$$\sup\{|x - y| : x, y \in I_n\} = \frac{C}{2^n},$$

where C does not depend on n .

(a) Show that there exists $x \in \mathbb{R}$ such that $\bigcap_{n=1}^{\infty} I_n = \{x\}$.

(b) Let r be a positive real number. Show that there exists n such that $I_n \subset N_r(x)$.

H3. Let K be a compact subset of a metric space X . Show that K contains a dense subset which is (finite or) countable.

H4. Let E be a subset of $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^4 + z^6 < 5\}$. Show that if E contains all its limit points, then E is compact.