Math 140A, Winter 2009. Homework 8. Due Wednesday March 4.

Rudin Chapter 4, problems 1,2,3,4.

H1. Calculate the smallest constant C so that for all points $y = (y_1, \ldots, y_k) \in \mathbb{R}^k$,

$$\left(\sum_{i=1}^{k} |y_i|^2 \right)^{1/2} \leq C \sup_{1 \leq i \leq k} |y_i|.$$

Demonstrate that your choice is correct.

H2. A **norm**, $|\ |$ on $\mathbb{R}^k$ is a function from $\mathbb{R}^n$ to $[0, \infty)$ which satisfies:

1. $|x| \geq 0$ for every $x \in \mathbb{R}^k$, and $|x| = 0$ if and only if $x = 0$.
2. $|\lambda x| = |\lambda||x|$ for all $x \in \mathbb{R}^k$ and real numbers λ.
3. $|x + y| \leq |x| + |y|$ for all $x, y \in \mathbb{R}^k$.

(a). Show that if $|\ |$ is a norm on $\mathbb{R}^k$, then

$$d(x, y) = |x - y|$$

defines a metric on $\mathbb{R}^k$.

(b). Show that if $|\ |$ and $\|\ \|$ are two norms on $\mathbb{R}^k$ and if

$$|x| \leq C\|x\|, \qquad \text{for all } x \in \mathbb{R}^k,$$

then for any $r \geq 0$,

$$\{x : \|x\| \leq r\} \subset \{x : |x| \leq Cr\}.$$