MATH 140A, WINTER 2008. MIDTERM 1.

1. (a). (5pt) Suppose X is a subset of $\mathbb{R}$. Define the supremum of X.

(b). (30pt) Suppose that X is a subset of the positive real numbers which is bounded above. Set

$$Y = \{x^2 : x \in X\}.$$

Show that

$$\sup Y \;=\; (\sup X)^2.$$

(c) (10pt). By defining the set X appropriately, use your answer to (b) to show that there exists a positive real number x with $x^2 = 3$.

2. (30pt) For this problem you may use the fact that a set X is countable if and only if there exists a sequence $x_1, x_2, \ldots$ whose range is X. However if you choose to use other results you should prove them.

Show that the rational numbers in the interval $[0, 1]$ are countable.

3.(25pt) Decide whether the following statements are true or false. You need not prove true statements. However, if the statement is false then give a counterexample.

(a). A countable intersection of open subsets of $\mathbb{R}$ is open.

(b). If F is a function and A, B are subsets of the domain of F then $F(A \cup B) = F(A) \cup F(B)$.

(c). If A is a countable subset of an uncountable set B, then $B - A$ is uncountable.

(d). If S is an infinite subset of $\mathbb{R}$ then S has an accumulation point in $\mathbb{R}$.

(e). If S is an uncountable subset of $\mathbb{R}$ then S has an accumulation point in $\mathbb{R}$.