MATH 140A, WINTER 2008. MIDTERM 2.

Problem 1 (30%). (a). Define what it means for a sequence (x_n) in the metric space (M, d) to converge.

(b). Show that if (x_n) converges to x and (y_n) converges to y in (M, d), then $d(x_n, y_n)$ converges to $d(x, y)$ in $\mathbb{R}$.

Problem 2 (40%). Naming any tests you use, prove the convergence or divergence of

(a). The series $\sum_{n=1}^{\infty} \frac{x^n}{n}$, where x is real.

(b). The series $\sum_{n=1}^{\infty} \left(n^{(1/n^2)} - 1\right)$.

Problem 3 (30%). Decide whether the following statements are true or false. You need not prove true statements. However, if the statement is false then give a counterexample.

(a). If A is a bounded subset of $\mathbb{R}^2$, then every open cover of A has a finite subcover.

(b). Suppose (M, d) is a metric space and S is a subset of M. If $A \subset S$ and A is open in the metric subspace S, then A is also open in the metric space M.

(c). If the complex sequence (a_n) is Cauchy, then the partial sums s_n of the series $\sum_{n=1}^{\infty} a_n$ also form a Cauchy sequence.

(d). Suppose (x_n) and (y_n) are real sequences. If $x_n \to \infty$ as $n \to \infty$, and $y_n \to 0$ as $n \to \infty$, then $x_n \cdot {y_n}^n \to 0$ as $n \to \infty$.

(e). If (a_n) is a positive sequence which is not necessarily decreasing, and if $\sum a_n$ converges, then $\sum 2^k a_{2^k}$ converges.