MATH 140A, WINTER 2008. PRACTICE MIDTERM 2.

1. Prove that a sequence (x_n) in a metric space (M, d) can converge to at most one point.

2. Using any tests you need (including the binomial theorem) prove the convergence or divergence of

(a). The sequence $n^{1/n}$.

(b). The series $\sum_{n=1}^{\infty} n!\, x^{(n^2)}$ where $x > 0$.

3. Decide whether the following statements are true or false. You need not prove true statements. However, if the statement is false then give a counterexample.

(a). If a metric space (M, d) is closed and bounded then every infinite subset of M has an accumulation point in M.

(b). If (x_n) is a sequence in a metric space (M, d), then the range $\{x_n\}$ can have at most one accumulation point.

(c). If the real sequences (x_n) and (y_n) converge, then the sequence $((x_n, y_n))$ converges in $\mathbb{R}^2$.

(d). Every compact metric space is bounded.

(e). If a real sequence (a_n) converges to zero then the series $\sum_{n=1}^{\infty} a_n$ converges.

(f). If a series $\sum_{n=1}^{\infty} b_n$ converges and if the positive sequence $(|b_n|)$ is decreasing then $\sum_{n=1}^{\infty} |b_n|$ converges.

(g). If the real sequence (b_n) diverges to infinity then $b_n^{1/(n^2)}$ also diverges to infinity.

(h). If the complex sequences (w_n) and (z_n) converge then so does w_n/z_n.

(i). If (a_n) converges to zero then $\sum \frac{a_n}{n}$ converges.