MATH 140B. HOMEWORK 3. KATE OKIKIOLU

Reading: Rudin 57–69.

H1. Show that the real sequences $\{a_n\}$ and $\{b_n\}$ converge as $n \to \infty$ if and only if the complex sequence $\{z_n\}$ converges, where $z_n = a_n + ib_n$.

H2. Suppose that f is differentiable on $[a, b]$. Then f is strictly increasing on $[a, b]$ if $f' > 0$ on $[a, b]$.

(a) Give a differentiable function f which is strictly increasing on $[-1, 1]$ such that $f'(0) = 0$.

(b) Exactly what can you say about f' if f is strictly increasing on $[a, b]$? (hint: to make the problem easier, assume that f' is continuous.)

H3. Prove the ratio test: If $\{b_n\}$ is a complex sequence such that $|b_{n+1}/b_n|$ converges to ρ, then:

If $\rho < 1$ then $\sum b_n$ converges, while if $\rho > 1$ then b_n does not tend to zero.

H4. Prove the binomial theorem.