MATH 140B. HOMEWORK 6.

For a real number t, define real numbers $\cos t$ and $\sin t$ by

$$e^{it} \;=\; cost \;+\; i \sin t.$$

We see that $\cos 0 = 1$ and $\sin 0 = 0$.

H1. Using the properties for the exponential function which we have already developed, show that

(i)
$$\sin^2 t \;+\; \cos^2 t \;=\; 1,$$

(ii)
$$\frac{d \cos t}{dt} \;=\; -\sin t, \qquad \frac{d \sin t}{dt} \;=\; \cos t.$$

(iii)
$$\sin(a+b) \;=\; \sin a \cos b \;+\; \sin b \cos a, \qquad \cos(a+b) = \cos a \cos b - \sin a \sin b.$$

H2. (a) Suppose that $\cos t$ never vanishes on $[0, \infty)$. Show that it must be strictly decreasing, and hence converges to some value a as $t \to \infty$. What can you say about a? Use the mean value theorem to get a contradiction.

(b) Let p denote the infimum of the set of positive numbers t with $\cos(t/2) = 0$. Why is it true that $\cos p/2 = 0$?

(c) Show that the exponential function is periodic with period $2\pi i$.

(d) For which values a is the line $t = a$ a line of reflectional symmetry of the graph $y = \cos t$? Prove this.