

MATH 140B, SPRING 2008.

1.(30%) Set

$$f(x) = \frac{2}{3} \left(x + \frac{1}{x^2} \right).$$

(a). Prove that $f([1, \infty)) \subset [1, \infty)$.

(b). Using the mean value theorem or otherwise, show that f is a contraction of $[1, \infty)$, and give a value for the contraction constant.

(c). Define x_n recursively by $x_0 = 1$, $x_j = f(x_{j-1})$. Calculate explicit constants C and α so that

$$\left| x_j - 2^{1/3} \right| < C\alpha^j.$$

2.(30%) Calculate

(a)
$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \left(1 + \frac{x}{2}\right)}{x^2}$$

(b)
$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} + \frac{1}{n^2} \right)^n.$$

3.(30%) What can you tell me about the power series

(a)
$$\sum_{n=0}^{\infty} \frac{n^n}{n!} z^n.$$

(b)
$$\sum_{n=1}^{\infty} (\log n) z^n$$

4. (10%) Make up a question which is not in the book.