# HOMEWORK 6

Due Monday November 12, 2001

Do Carmo Section 3.2 questions 2, 5, 7, 8, 9, 13.
Plus the following question:

Consider the sphere $S^2$ given by $x^2 + y^2 + z^2 = 1$. This is parameterized by

$$\mathbf{r} : (\theta, \phi) \ \rightarrow \ (\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi).$$

(a). Calculate the bases for $T_{\mathbf{p}}S^2$ associated to this parameterization at the points $\mathbf{p} = \mathbf{r}(0, \pi/2)$, and $\mathbf{q} = \mathbf{r}(\alpha, \pi/2)$.
Let $F$ denote a rotation of $S^2$ by angle $\theta$ about the $z$-axis. In the parameterization $\mathbf{r}$ this is given by

$$(\theta, \phi) \ \rightarrow \ (\theta + \alpha, \phi).$$

(b). The vector $\mathbf{w} = (0, 3, -2)$ is tangent to $S^2$ at $\mathbf{p}$. Write $\mathbf{w}$ in coordinates relative to the basis for $T_{\mathbf{p}}S^2$ found in (a).
(c). Write down the matrix for $dF_{\mathbf{p}}$ relative to the bases found in (a). Hence calculate the coordinates of $dF_{\mathbf{p}}\mathbf{w}$ relative to the basis for $T_{\mathbf{q}}S^2$ found in (a).
(d). Use (c) to calculate $dF_{\mathbf{p}}\mathbf{w}$.
(e). Let $f$ be a rotation of $\mathbb{R}^3$ through angle $\alpha$ about the $z$-axis. Then $f$ is a linear map. calculate the matrix of $f$ and hence write down the vector $f(x, y, z)$.
(f). Calculate the matrix of $df_{\mathbf{p}}$. Hence calculate $df_{\mathbf{p}}\mathbf{w}$.
You should find that $dF_{\mathbf{p}}\mathbf{w} = df_{\mathbf{p}}\mathbf{w}$.