LECTURES 17: GAUSS THEOREM.

November 26, 2001

Today's Quiz: Suppose that a plane has equation $\mathbf{r} \cdot \mathbf{N} = 0$, where $\mathbf{N}$ is the unit normal. If $\mathbf{w}$ is a vector in $\mathbb{R}^3$. Calculate the orthogonal projection of $\mathbf{w}$ onto the plane?

Last Time:

$$II(\mathbf{u}, \mathbf{v}) = \langle -d\mathbf{N}(\mathbf{u}), \mathbf{v} \rangle.$$

$$\mathbf{r} : U \to S$$

is a parameterization. The matrix of the second fundamental form in the basis $\mathbf{r}_u, \mathbf{r}_v$:

$$\begin{pmatrix} e & f \\ f & g \end{pmatrix} = \begin{pmatrix} \langle \mathbf{N}, \mathbf{r}_{uu} \rangle & \langle \mathbf{N}, \mathbf{r}_{uv} \rangle \\ \langle \mathbf{N}, \mathbf{r}_{vu} \rangle & \langle \mathbf{N}, \mathbf{r}_{vv} \rangle \end{pmatrix}.$$

The matrix of dN is

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = -\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} e & f \\ f & g \end{pmatrix}.$$

Gauss curvature

$$K = \det d\mathbf{N} = (EG - F^2)(eg - f^2).$$

Theorem. K can be written in terms of E, F, G. (Hence it is invariant under local isometries.)

Proof. We follow Do Carmo: we introduce the Christoffel symbols and show that they can be written in terms of E, F, G. Then we show that K can be written in terms of the Christoffel symbols.

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