

PRACTICE MIDTERM 1.

1. (a). Consider the parameterized ellipse

$$\mathbf{r}(t) = (2 \cos t, \sin t, 0).$$

Calculate the curvature at $t = 0$.

- (b). What is the torsion?

2. Decide which of the following are regular surfaces and explain your answer.

(a). $e^x = xyz$.

(b). $z^2 = x^2 + y^2$.

3. Show that $x^2 + y^2 - z^2 = 1$ is a regular surface and that

$$(t, \theta) \rightarrow (\sqrt{1+t^2} \cos \theta, \sqrt{1+t^2} \sin \theta, t), \quad 0 < \theta < 2\pi, \quad -\infty < t < \infty$$

is a parameterization.

4. Decide whether the map

$$\mathbf{r} : (x, y, z) \rightarrow \left(\frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right)$$

is smooth from the cylinder $x^2 + y^2 = 1$ to the sphere $x^2 + y^2 + z^2 = 1$. Explain.