

## PRACTICE MIDTERM 2.

1. Determine those points  $\mathbf{p}$  on the hyperbolic paraboloid  $H$  given by  $z = xy$ , such that the tangent plane  $T_{\mathbf{p}}H$  contains  $(0, 0, 0)$ .

2. Show that  $(-1, 0, 1)$  is tangent to the cylinder  $y^2 + z^2 = 1$  at  $(0, 1, 0)$ , and calculate  $dF_{(0,1,0)}(-1, 0, 1)$  when  $F$  is the map from the cylinder to the sphere  $x^2 + y^2 + z^2 = 1$  given by

$$F(x, y, z) = \left( \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right).$$

3. Let

$$(\rho(s), z(s)), \quad -\infty < s < \infty$$

be a regular curve parameterized by arclength with  $\rho > 0$ , and consider the surface of revolution

$$\mathbf{r}(s, \theta) = (\rho(s) \cos \theta, \rho(s) \sin \theta, z(s)), \quad -\infty < s < \infty, \quad -\pi < \theta < \pi.$$

(a). Calculate the matrix

$$\begin{pmatrix} E & F \\ F & G \end{pmatrix}$$

of first fundamental form in the parameterization  $\mathbf{r}$ .

(b). Suppose that for every curve  $\alpha : [0, 1] \rightarrow \mathbb{R} \times (-\pi, \pi)$ , the length of  $\alpha$  is the same as the length of  $\mathbf{r} \circ \alpha$ . What can you say about  $\rho(s)$  and  $z(s)$ ?

4. Consider the parameterization of the cone  $S$  given by

$$\mathbf{r}(\theta, z) = (z \cos \theta, z \sin \theta, z), \quad z > 0, \quad -\pi < \theta < \pi.$$

(a). Show that the vectors

$$\mathbf{e}_1 = \frac{1}{z} \frac{\partial \mathbf{r}}{\partial \theta}, \quad \mathbf{e}_2 = \frac{1}{\sqrt{2}} \frac{\partial \mathbf{r}}{\partial z}$$

form an orthonormal base for the tangent space of  $S$  at  $\mathbf{r}(\theta, z)$ .

(b). Calculate a smoothly varying unit normal  $\mathbf{N}$  at  $\mathbf{r}(\theta, z)$ .

(c). Calculate  $d\mathbf{N}(\mathbf{e}_1)$  and  $d\mathbf{N}(\mathbf{e}_2)$ . Hence calculate the the eigenvalues of  $-d\mathbf{N}$  (these are the principal curvatures).