Math 20E Vector Calculus: Homework 1

1.1. 16. Use set theoretic or vector notation to describe the line passing through $(-5, 0, 4)$ and $(6, -3, 2)$.

1.2. 8. Compute $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, $\mathbf{u} \cdot \mathbf{v}$ where $\mathbf{u} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

14. Find the projection of $\mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$ onto $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.
23. A force of 50lb is directed 50° above the horizontal pointing to the right. Determine its horizontal and vertical components.

1.3. 6. A triangle has vertices (0,0,0), (1,1,1), and (0,-2,3). Find its area.

7. What is the volume of the parallelopiped with sides $2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $5\mathbf{i} - 3\mathbf{k}$, and $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$?

12. Describe all unit vectors orthogonal to both the given vectors: $2\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$, $-4\mathbf{i} + 8\mathbf{j} - 6\mathbf{k}$.
16a. Find an equation for a plane which passes through $(0, 0, 0)$, $(2, 0, -1)$ and $(0, 4, -3)$.
1.5. 7. Compute AB, $\det A$, $\det B$, $\det AB$ and $\det A + B$ for

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 3 & 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} -2 & 0 & 2 \\ -1 & 1 & -1 \\ 1 & 4 & 3 \end{pmatrix}.$$

17 (not assigned but needed for 18). Verify the inverse of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\dfrac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

18. Use your answer to Exercise 17 to show that the solution of the system

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix}.$$

Review Ch. 1. 29. (a). Show that for two $n \times n$ matrices A and B, and $\mathbf{x} \in \mathbb{R}^n$,

$$(AB)(\mathbf{x}) = A(B\mathbf{x}).$$

(b). What does the equality in part (a) imply about the relationship between the composition of the mappings $\mathbf{x} \to A\mathbf{x}$, $\mathbf{y} \to B\mathbf{y}$, and matrix multiplication?

Math 20E Vector Calculus: Homework 2

1.4. 4(a). Describe the surfaces r =constant, θ =constant and z =constant in the cylindrical coordinates system.
(b). Describe the surfaces ρ =constant, θ =constant, and ϕ =constant in the spherical coordinate system.

2.1. 6. Draw the level curves $f(x, y) = c$ in the xy plane and sketch the graph of the function $f(x, y) = (x^2 + y^2)^{1/2}$, with $c = 0, 1, 2, 3, 4, 5$.

11. Sketch or describe the level surfaces and a section of the graph of the function $f(x, y, z) = -x^2 - y^2 - z^2$.

21. Sketch or describe the surface in $\mathbb{R}^3$ of the equation $z^2 = y^2 + 4$.

28. Sketch or describe the surface in $\mathbb{R}^3$ of the equation $y^2 = x^2 + z^2$.

2.3. 6(a). Compute the tangent plane to the graph of $f(x, y)$ at the point $(0, 0)$, where $f(x, y)$ is
(a). xy.
(b). e^{xy}.
(c). $x \cos x \ \cos y$.
(d). $(x^2 + y^2) \log(x^2 + y^2)$.

8. Compute the matrix of partial derivatives of the functions
(c). $f(x, y) = (x + y, x - y, xy)$,
(d). $f(x, y) = (x + z, y - 5z, x - y)$.

12(c). Use the linear approximation to approximate a suitable function $f(x, y)$ and thereby estimate

$$\sqrt{(4.01)^2 \ + \ (3.98)^2 \ + \ (2.02)^2}.$$

18. Evaluate the gradient of $f(x, y, z) = \log(x^2 + y^2 + z^2)$ at $(1, 0, 1)$. medskip
20. Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is a linear map. What is the derivative of f?

2.4. 8. Determine the velocity vector of the path $\mathbf{r}(t) = (4e^t, 6t^4, \cos t)$.

18. Suppose that a particle following the path $\mathbf{c}(t) = (e^t, e^{-t}, \cos t)$ flies off on a tangent at time $t = 1$. Compute the position of the particle at time $t = 2$.

2.5. 4. Verify the chain rule for $\partial h/\partial x$ where $h(x, y) = f(u(x, y), v(x, y))$ and

$$f(u, v) = \frac{u^2 + v^2}{u^2 - v^2}, \qquad u(x, y) = e^{-x-y}, \qquad v(x, y) = e^{xy}.$$

5(c). Verify the special case of the chain rule for the composition $f \circ \mathbf{c}$ when

$$f(x, y) = (x^2 + y^2) \log \sqrt{x^2 + y^2}, \qquad \mathbf{c}(t) = (e^t, e^{-t}).$$

15. Let $f : \mathbb{R}^2 \to \mathbb{R}^2 : (x, y) \to (e^{x+y}, e^{x-y})$. Let $\mathbf{c}(t)$ be a path with $\mathbf{c}(0) = (0, 0)$ and $\mathbf{c}'(0) = (1, 1)$. What is the tangent vector to the image of $\mathbf{c}(t)$ under f at $t = 0$?