Math 20E. Midterm 1. April 25, 2004
ANSWER ALL 5 QUESTIONS. Each question is worth 20 points.

1. Let C be the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 2)$, oriented so that when you trace out the triangle, you pass through the points in the order given. Compute the circulation $\int_C \mathbf{F} \cdot d\mathbf{R}$ where $\mathbf{F} = x\mathbf{i} + z\mathbf{j} - y\mathbf{k}$.

2. For the following vector fields $\mathbf{F}$, either find a scalar field ϕ such that $\mathbf{F} = \nabla\phi$, or show that $\mathbf{F}$ is not conservative on $\mathbb{R}^3$.

(a). $\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$.

(b). $\mathbf{F} = yz\mathbf{i} + (xz + z)\mathbf{j} + (xy + y + 1)\mathbf{k}$.

3. (a). Calculate the equation of the flow line for the vector field $\mathbf{F} = e^{-x}\mathbf{i} + e^{x}\mathbf{j} + \mathbf{k}$ which passes through the point (x_0, y_0, z_0).

(b). Determine which, if any, of the following three points lie on the same flow line.

$$(0, 0, 0) \qquad (1, 1, 1) \qquad \left(1, \tfrac{e^2-1}{2}, e - 1\right)$$

4. For a real valued constant α, let

$$\mathbf{F}(x, y, z) = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^\alpha}.$$

(a). Determine for which values of α the divergence $\nabla \cdot \mathbf{F}$ is positive.

(b). In a region which contains the point $P = (1, 1, 0)$, a fluid has flow

$$\frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^2}.$$

No fluid is being created or removed from the system. Is the density of the fluid increasing or decreasing at P?

5. (a). Calculate the second order Taylor polynomial of $f(x, y) = y^x = e^{x \ln y}$ based at the point $(x_0, y_0) = (0, 1)$.

(b). Use your answer to (a) to write down an estimate for $0.9^{0.1}$.