Math 20E. Midterm 1. February 6, 2005
ANSWER ALL 5 QUESTIONS. Each question is worth 20 points.

1. $\mathbf{F}(x,y) \;=\; \left(\dfrac{y}{x}\;,\; x^2+y^2\right)$.

(a). Compute the matrix $D\mathbf{F}$ at the point (x,y).

(b). Let $\mathbf{c}(t) = (x(t), y(t))$ be a curve with $\mathbf{c}(0) = (1,1)$ and $\mathbf{c}'(0) = (-1,2)$. Use the chain rule to compute $(\mathbf{F}\circ\mathbf{c})'(0)$.

2. $f(x,y) = e^{x\sin y}$.

(a). Calculate the second order Taylor polynomial of f based at the point $(1,0)$.

(b). Use your answer to (a) to write down an estimate for $e^{0.9\sin 0.2}$.

3. $\mathbf{F}(x,y,z) = \left(\dfrac{x}{r}, \dfrac{-y}{r}, 0\right)$, where $r = \sqrt{x^2+y^2}$.

(a). Calculate $\nabla\cdot\mathbf{F}(x,y,z)$.

(b). Suppose $\mathbf{F}$ represents the velocity field for a gas. Is a small region of gas located near $(1,0,0)$ expanding or contracting?

4. The gradient of the scalar field $f(x,y,z)$ is $\nabla f \;=\; \left(\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z}\right) \;=\; (f_x, f_y, f_z)$.

(a). Calculate $\nabla\times\nabla f$.

(b). Calculate $\nabla\times(y\mathbf{i})$.

(c). Does there exist a scalar field f with $\nabla f = y\mathbf{i}$?

5. $f = xy + z^2$.

(a). Calculate the direction in which f increases fastest at the point $(1,1,1)$.

(b). Find, if possible, a constant a so that the curve $\mathbf{c}(t) = (ae^t, ae^t, e^{at})$ is a flow line for the vector field ∇f.