

Math 20E. Midterm 2. May 17, 2004
ANSWER ALL 4 QUESTIONS.

1. (25%) Let $\mathbf{F} = e^{x-y} \mathbf{i} + (z - e^{x-y}) \mathbf{j} + y \mathbf{k}$.

(a). Determine whether \mathbf{F} is conservative.

(b). Calculate $\int_C \mathbf{F} \cdot d\mathbf{R}$ where C is the parameterized curve given by

$$x = \tan\left(\frac{\pi t}{4}\right), \quad y = \ln(1 + (e - 1)t), \quad z = t^3, \quad 0 \leq t \leq 1.$$

(Hint: Use your answer to part (a).)

2. (20%) Calculate a vector potential for $\mathbf{F} = (x + y)\mathbf{i} - y\mathbf{j}$.

3. (25%) By substituting $u = xy$ and $v = y/x$, calculate the area of the region S given by

$$1 \leq xy \leq 2, \quad \frac{1}{2} \leq \frac{y}{x} \leq 2, \quad (x > 0, y > 0).$$

(You may assume that the mapping $(x, y) \rightarrow (u, v)$ is one-to-one from the quadrant $x > 0, y > 0$ onto the quadrant $u > 0, v > 0$.)

4. (30%) Let $\mathbf{F} = \mathbf{j}$. For the following choices of S and \mathbf{n} , calculate $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$.

(a). S is the piece of the plane $z = y + 2$ with $x^2 + y^2 \leq 4$, and \mathbf{n} is the unit normal with $\mathbf{n} \cdot \mathbf{k} > 0$.

(b). S is the piece of cylinder $x^2 + y^2 = 4$ with $-2 \leq z \leq y + 2$, and \mathbf{n} is the outward unit normal.