MATH 20E, MIDTERM 2, Spring 1998, Kate Okikiolu

1. (10%) The directional derivative of $g(x,y,z) = e^x yz^2$ in the direction $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ at $(0,1,1)$ is

A). $\sqrt{6}$.
B). 1.
C). 7/3.
D). e.
E). None of the above.

Figure 1 shows the level curves of a function $f(x,y)$ in the plane.

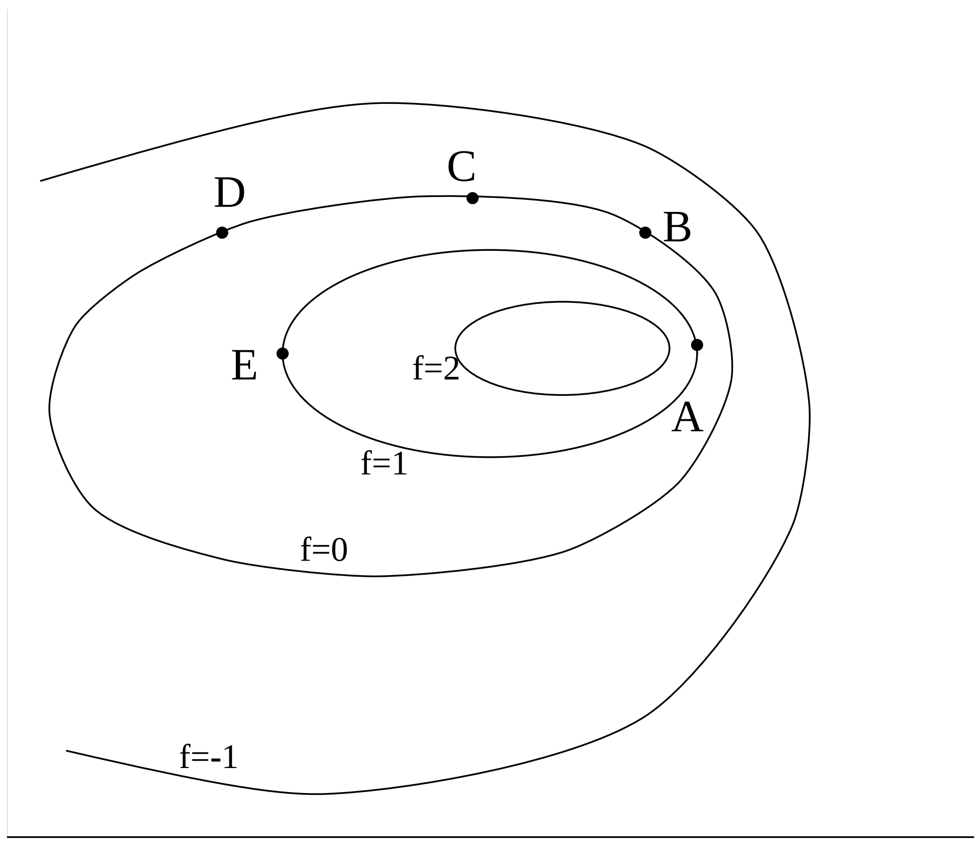

Figure 1

2. (10%) At which one of the points **A, B, C, D, E** is ∇f parallel to $\mathbf{j}$?

3. (10%) At which one of the points **A, B, C, D, E** is $|\nabla f|$ largest?

4. (10%) At which one of the points **A, B, C, D, E** is f increasing in the direction $\mathbf{i}+\mathbf{j}$?

5. (10%) At which one of the following points is the vector $\mathbf{j} + \mathbf{k}$ normal to the parabaloid

$$z = \frac{x^2}{2} + \frac{y^2}{2} + x?$$

A). $(-1, 1, 0)$.
B). $(-1, -1, 0)$.
C). $(0, 2, 1)$.
D). $(0, 1, 1)$.
E). $(0, 0, 0)$.

6. (10%) $\mathbf{F}(x, y, z)$ and $\mathbf{G}(x, y, z)$ are vector fields. Which of the following expressions does not make sense?
A). $\nabla \times (\nabla \times \mathbf{F})$.
B). $\nabla(\nabla \cdot (\nabla \times \mathbf{F}))$.
C). $\nabla \cdot (\nabla \cdot (\nabla \times \mathbf{F}))$.
D). $\nabla \times (\mathbf{F} \nabla \cdot \mathbf{G})$.
E). $\nabla \cdot (\mathbf{F} \times \mathbf{G})$.

7. (10%) $\mathbf{F}$ represents the flow of a fluid in space. Which of the following statements is true?
A). $\nabla \cdot \mathbf{F}$ measures the extent to which the fluid is swirling around.
B). $\nabla \cdot \mathbf{F}$ can never be zero.
C). If the density of the fluid is constant and there are no sources or sinks then $\nabla \cdot \mathbf{F} = 0$.
D). If the flowlines are straight lines then $\nabla \cdot \mathbf{F} = 0$.
E). If $\nabla \cdot \mathbf{F} = 0$ everywhere it means that the molecules of fluid are not moving.

8. (10%) For which points is

$$\nabla \times (x^2 \mathbf{j} + yz\mathbf{k})$$

parallel to $\mathbf{i}$?
A). At all points, since $\nabla \times \mathbf{F}$ is always perpendicular to $\mathbf{F}$.
B). On the plane $x = 0$.
C). Whenever $yz = 0$.
D). On the surface $x^2 + yz = 1$.
E). Never.

The vector field $\mathbf{F}(x, y, z)$ has the form $f_1(x, y)\mathbf{i} + f_2(x, y)\mathbf{j}$ and is shown in Figure 2.

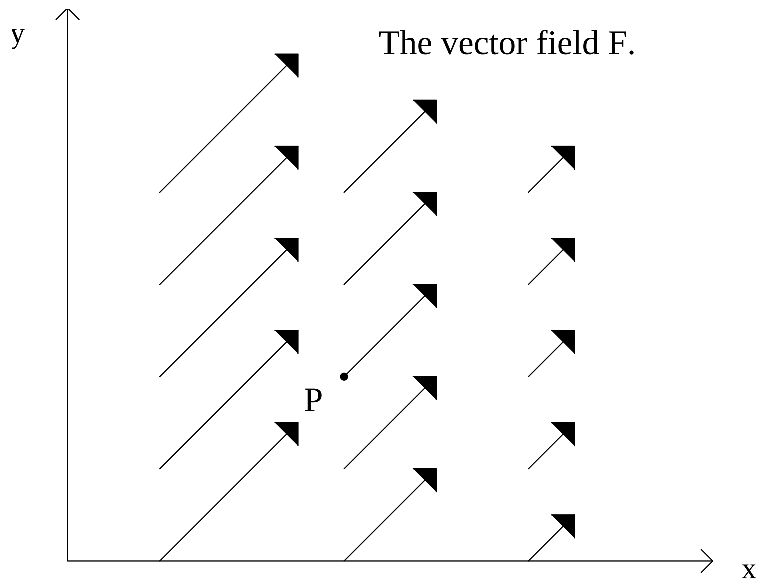

Figure 2

9. (10%) For the vector field $\mathbf{F}$ in Figure 2, decide which one of the following statements can be true.
A). $\mathbf{F}$ is the vector field

$$\frac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}.$$

B). $\mathbf{F}$ is the vector field $-y\mathbf{i} + x\mathbf{j}$.
C). The flow lines of $\mathbf{F}$ are circles.
D). $\mathbf{F} \cdot \mathbf{F} = 1$ at every point.
E). The flow lines of $\mathbf{F}$ are straight lines.

10. (10%) By computing the signs of

$$\frac{\partial F_1(P)}{\partial x}, \quad \frac{\partial F_2(P)}{\partial x}, \quad \frac{\partial F_1(P)}{\partial y}, \quad \frac{\partial F_1(P)}{\partial z},$$

Decide which of one the following statements can be true.
A). $(\nabla \times \mathbf{F})(P) \cdot \mathbf{k} > 0$ and $\nabla \cdot \mathbf{F}(P) > 0$.
B). $(\nabla \times \mathbf{F})(P) \cdot \mathbf{k} < 0$ and $\nabla \cdot \mathbf{F}(P) < 0$.
C). $(\nabla \times \mathbf{F})(P) \cdot \mathbf{k} > 0$ and $\nabla \cdot \mathbf{F}(P) < 0$
D). $(\nabla \times \mathbf{F})(P) \cdot \mathbf{k} < 0$ and $\nabla \cdot \mathbf{F}(P) > 0$.
E). $(\nabla \times \mathbf{F})(P) = \mathbf{i}$.