

1. Let $\phi = x^2 - z$ and let $\psi = \frac{1}{x} + z$.

Find a direction \mathbf{B} so that ϕ and ψ both *increase* when you move away from the point $(1, 1, 1)$ in the direction \mathbf{B} .

(8 points.)

2. **(12 points total.)** Consider the vector field $\mathbf{F} = \frac{y\mathbf{i}}{x^2 + y^2} - \frac{x\mathbf{j}}{x^2 + y^2}$ in the plane.

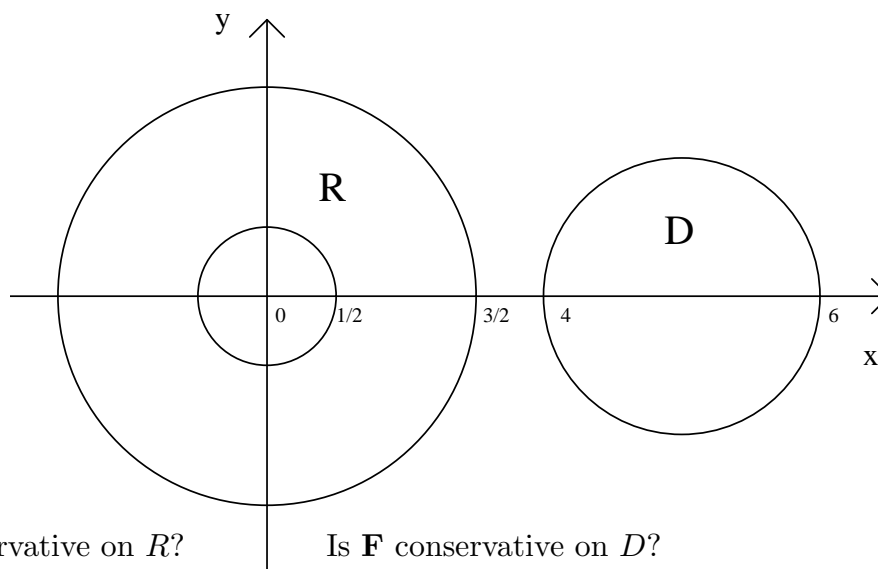
a). Calculate $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.

(4 points.)

b). Calculate $\int_C \mathbf{F} \cdot d\mathbf{R}$, where C is the circle with center $(0, 0)$ and radius 1 directed anticlockwise.

(4 points.)

Consider the annulus R and the disc D shown below.



c). Is \mathbf{F} conservative on R ?

Is \mathbf{F} conservative on D ?

3. (20 points total.)

$$\mathbf{F} = \frac{x}{x^2 + z^2}\mathbf{i} + y\mathbf{j} + \frac{z}{x^2 + z^2}\mathbf{k}.$$

a). Let C be the flow line of \mathbf{F} through the point $(2, e, 2)$. Parameterize C with x as the parameter.

(8 points.)

b). Find a scalar field ϕ with $\nabla\phi = \mathbf{F}$ and $\phi(2, e, 2) = 0$.

(8 points.)

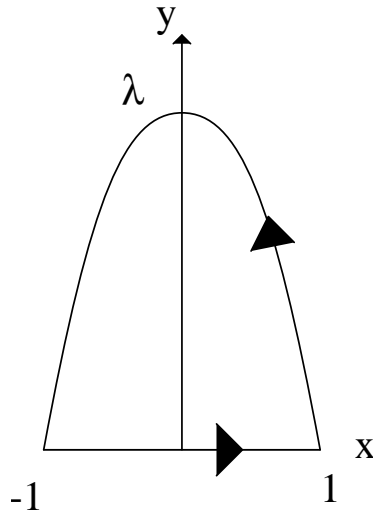
c). What is the angle between the surface $\phi = 0$ and the curve C from part a), at the point $(2, e, 2)$?

(4 points.)

4.(20 points total.) C is the closed curve which consists of the interval from $(-1, 0)$ to $(1, 0)$ together with the arc of the parabola

$$y = \lambda(1 - x^2), \quad -1 \leq x \leq 1,$$

traversed anticlockwise. Here, λ is just a constant. D is the region bounded by the curve C .



$$\mathbf{F} = x^3 \mathbf{j}.$$

a). Calculate $\int_C \mathbf{F} \cdot d\mathbf{R}$.
(10 points.)

b). Calculate $\nabla \times \mathbf{F}$.
(2 points.)

c). Without using your answer to a), calculate $\int_D \nabla \times \mathbf{F} \cdot \mathbf{k} \, dA$.
(8 points.)