Math 20E Midterm 2, Fall 98, Lindblad.

1. Let S be the triangle with vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ oriented so that the normal $\mathbf{n}$ points in the direction out from the origin. Let C be the boundary curve of S oriented counterclockwise seen from the side of the normal. Let $\mathbf{F} = (x+y)\mathbf{i} + (y+z)\mathbf{j} + (z+x)\mathbf{k}$.

a) Find the flux of $\mathbf{F}$ out from S: $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$.

b) Find the line integral of $\mathbf{F}$ around C: $\int_C \mathbf{F}\cdot\mathbf{dR}$

2. Let R be the 3-dimensional region $R = \{x^2/4 + y + z^2/4 \le 1,\ y \ge 0\}$. Let S be the surface of R with the normal oriented outwards. Note that S has two parts $\{x^2/4 + y + z^2/4 = 1,\ y \ge 0\}$ and $\{y = 0,\ x^2/4 + z^2/4 \le 1\}$.

a) Find the area of S.

b) Find the flux of $\mathbf{F} = x\mathbf{i} - y\mathbf{j} + \mathbf{k}$ through S; $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$.

3. Find $\iint_R x\,dxdy$, where R is the region bounded by $y = \dfrac{1}{x^2}$, $y = \dfrac{2}{x^2}$, $y = x$ and $y = 2x$ by making the change of variables $u = x^2 y$ and $v = y/x$.

4. Let S be the part of the hyperboloid $x^2 + y^2 - z^2 = 1$ with $0 \le z \le 1$.
A parametrization of the surface is given by

$$\mathbf{R}(u,v) = (\cos u - v\sin u)\mathbf{i} + (\sin u + v\cos u)\mathbf{j} + v\mathbf{k}, \qquad 0 \le u \le 2\pi, \quad 0 \le v \le 1.$$

a) Find the area element dS expressed in terms of the parametrization $du\,dv$.

b) Find the surface integral $\iint_S z\,dS$.