Math 20E Midterm 2, Winter 05, Lindblad.

1. (a) Let $\mathbf{F} = \mathbf{i} + \mathbf{j}$ and let $\mathbf{c}_1$ be the curve $\mathbf{c}_1 = \cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\,\mathbf{k}$, $0 \le t \le \pi/2$. Find the line integral $\displaystyle\int_{\mathbf{c}_1} \mathbf{F} \cdot d\mathbf{s}$.

(b) Let $\mathbf{F} = \mathbf{i} + \mathbf{j}$ and let $\mathbf{c}_2$ be the straight line segment from $(1, 0, 0)$ to $(0, 2, \pi/2)$. Find the line integral $\displaystyle\int_{\mathbf{c}_2} \mathbf{F} \cdot d\mathbf{s}$.

2. Find $\displaystyle\iint_R \frac{x - y}{(x + 2y)^2}\, dxdy$, where $R = \{(x, y);\ 2 \le x + 2y \le 4,\ 0 \le x - y \le 3\}$, by making a change of variables.

3. Let S is the closed surface of the region $W = \{(x, y, z);\ x^2 + y^2 + 2 \le z \le 6\}$, i.e. S is the surface consisting of the two parts S_1 and S_2, where $S_1 = \{(x, y, z);\ z = x^2 + y^2 + 2,\ x^2 + y^2 \le 4\}$, $S_2 = \{(x, y, z);\ z = 6,\ x^2 + y^2 \le 4\}$.

Find the flux of $\mathbf{F} = -\mathbf{k} + x\mathbf{i}$ out through S; $\displaystyle\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$.
Here $\mathbf{n}$ is the unit normal oriented out from W.

4. Let S be the surface given by the parametrization

$$\mathbf{T}(u, v) = v\,\cos u\,\mathbf{i} + v\,\sin u\,\mathbf{j} + (1 + v)\,\mathbf{k}, \qquad 0 \le u \le 2\pi, \quad 1 \le v \le 2.$$

a) Find the area element dS expressed in terms of the parametrization $du\,dv$.

b) Find the surface area using the parametrization above.