Name: ______________________________ **Section Number:** ______

TA Name: ______________________________ **Section Time:** ______

Math 20F.
Final Examination
June 10, 2005

1. (4 points) Find the solution to the matrix equation

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

and write it in parametric form.

#	Score
1	
2	
3	
4	
5	
6	
7	
8	
Σ	

2. (8 points) The matrices $A = \begin{bmatrix} 3 & 1 & 11 & 8 \\ 2 & 1 & 9 & 5 \\ 0 & -1 & -5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent.

(a) Explain why the nonzero rows of B form a basis for $\text{Row}(A)$, the row space of A.

(b) Find a basis for $\text{Col}(A)$, the column space of A.

(c) Find a basis for $\text{Nul}(A)$, the null space of A.

(d) Find a basis for $\text{Row}(A)^{\perp}$, the orthogonal complement of the row space of A. Be sure to explain how you know that it is a basis for $\text{Row}(A)^{\perp}$.

3. (4 points) The set $\mathcal{B} = \{1,\ 1+2t,\ 1+2t+4t^2\}$ is a basis for $\mathbb{P}_2$, the vector space of polynomials of degree at most two. The polynomial $\mathbf{p} = 1+4t^2$ is in $\mathbb{P}_2$. Find $[\mathbf{p}]_{\mathcal{B}}$, the coordinate vector for $\mathbf{p}$ with respect to the basis $\mathcal{B}$.

4. (4 points) Let $A = \begin{bmatrix} -1 & 2 & 10 \\ 2 & 1 & 10 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$.

Find an orthogonal basis for $\mathrm{Col}(A)$, the column space of A.

5. (8 points) Let W be a subspace of $\mathbb{R}^n$ with an orthogonal basis $\mathcal{B}_W = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p\}$, and let $\mathcal{B}_{W^\perp} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q\}$ be an orthogonal basis for $W^\perp$, the orthogonal complement of W.

(a) Explain why $\mathcal{S} = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q\}$ is an orthogonal set.

(b) Explain why the set $\mathcal{S}$ spans $\mathbb{R}^n$.

(c) Explain why $\mathcal{S}$ is linearly independent.

(d) Explain why $\dim W + \dim W^\perp = n$.

6. (8 points) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$.

(a) Find the least-squares solution $\hat{\mathbf{x}}$ to the matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) Verify that $\mathbf{b} - A\hat{\mathbf{x}}$ is in $\text{Col}(A)^{\perp}$, the orthogonal complement of the column space of A.

7. (8 points) Consider the matrix $A = \begin{bmatrix} 5 & 2 & -4 \\ 2 & 8 & 2 \\ -4 & 2 & 5 \end{bmatrix}$. The characteristic polynomial of A is $p(\lambda) = -\lambda(9-\lambda)^2$.

(a) Determine the eigenvalues of A.

(b) Find an orthogonal basis for each eigenspace of A.

(c) Find an orthogonal matrix P which orthogonally diagonalizes A.

8. (10 points) For each statement, circle **T** if it is *always* True; circle **F** if it is *ever* False. Please note that there will be a penalty for guessing incorrectly: 1 point will be assigned for each correct response, 0 points for each blank non-response, and 1 point will be *deducted* for each incorrect response. No justification is required.

(**T** **F**) If a square matrix has orthonormal columns, then it also has orthonormal rows.

(**T** **F**) If a matrix A is invertible and 1 is an eigen*value* of A, then 1 is an eigen*value* for A^{-1}.

(**T** **F**) Each eigen*vector* of an invertible matrix A is also an eigen*vector* of A^{-1}.

(**T** **F**) Each eigen*value* of an $n \times n$ matrix A is also an eigen*value* of A^2.

(**T** **F**) Each eigen*vector* of an $n \times n$ matrix A is also an eigen*vector* of A^2.

(**T** **F**) If A and B are invertible $n \times n$ matrices, then AB is similar to BA.

(**T** **F**) Row operations on a matrix A can change the linear dependence relations among the *rows* of A.

(**T** **F**) Row operations on a matrix A can change the linear dependence relations among the *columns* of A.

(**T** **F**) Row operations on a matrix A can change the null space.

(**T** **F**) If a system of linear equations has two different solutions, then it has infinitely many solutions.