

**Math 20F practice Midterm I Spring 06, Lindblad.**

1. (a) Determine if the following systems are consistent and if so give all solutions in parametric form:

$$(a) \quad \begin{aligned} x_1 - 2x_2 &= 3 \\ 2x_1 - 7x_2 &= 0 \\ -5x_1 + 8x_2 &= 5 \end{aligned} \quad (b) \quad \begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= 1 \\ -x_1 - x_2 + 4x_3 - x_4 &= 6 \\ -2x_1 - 4x_2 + 7x_3 - x_4 &= 1 \end{aligned}$$

2. Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ .

(a) Find the inverse of  $A$ .

(b) Find the solution to  $A\mathbf{x} = \mathbf{b}$ .

3. A linear transformation  $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$  satisfies  $T(\mathbf{e}_1) = (2, 1, 1)$  and  $T(\mathbf{e}_2) = (1, 1, 2)$ , where  $\mathbf{e}_1 = (1, 0)$  and  $\mathbf{e}_2 = (0, 1)$ .

(a) Find the standard matrix for  $T$ .

(b) Find  $T((-3, 4))$ .

(c) Is  $T$  one-to-one? Justify your answer.

(d) Is  $T$  onto? Justify your answer.

4.(a) Show that  $A = \begin{bmatrix} 1 & 2 & -2 & 0 & 7 \\ -2 & -3 & 1 & -1 & -5 \\ -3 & -4 & 0 & -2 & -3 \\ 3 & 6 & -6 & 5 & 1 \end{bmatrix}$  is row equivalent to  $B = \begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ .

(b) Find a basis for  $\text{Nul } A$ .

(c) Find a basis for  $\text{Col } A$ .