Math 20F Midterm II Spring 06, Lindblad.

1. Find the determinant of the following matrices

$$(a) \quad A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{bmatrix}, \qquad (b) \quad B = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}, \qquad (c) \quad B^4.$$

2. For each of the matrices below, either diagonalize it, i.e. find a real matrix P such that $P^{-1}AP$ etc. is diagonal, or explain why its not possible:

$$(a) \quad A = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}, \quad (b) \quad A = \begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix},$$

2. For each of the matrices below, either diagonalize it, i.e. find a real matrix P such that $P^{-1}AP$ etc. is diagonal, or explain why its not possible:

$$(a) \quad A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 5 \end{pmatrix}, \quad (b) \quad B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ -3 & 5 & 3 \end{pmatrix}.$$

3. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation which is the reflection in the line L given by $x_2 = x_1$.
(a) Find the eigenvalues and eigenvectors of T. Hint: Vectors on the line (s, s) are not changed by L whereas vectors perpendicular to the L, $(t, -t)$ are changed to $(-t, t)$.
(b) Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the two linearly independent eigenvectors of T found in (a). What is the matrix for T relative to the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$?
(c) What is the change of coordinate matrix from the $\mathcal{B}$ basis to the standard basis.
(d) What is the matrix for T relative to the standard basis?

4. Suppose a dynamical system is described by a difference equation $\mathbf{x}_{k+1} = A\mathbf{x}_k$, where $A = \begin{bmatrix} 1 & 1/2 \\ -1/4 & 1/4 \end{bmatrix}$.
(a) Find the eigenvalues and eigenvectors of A.
(b) Let $\{\mathbf{x}_k\}$ be the solution of the dynamical system with $\mathbf{x}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find an explicit formula for $\mathbf{x}_k$, for all k. What happens as $k \to \infty$?
(c) What does the largest eigenvalue tell you about the long term behavior of the system?