Math 231A Partial Differential Equations

Homework 2.

1. Section 2.5 number 1.

2. Section 2.5 number 2.

3. What is the definition of the function $f : \mathbb{R}^n \to \mathbb{R}$ being k times continuously differentiable?

4. We proved that if $U \subset \mathbb{R}^n$ is an open set and if $f : U \to \mathbb{R}$ is a Lipschitz function, then for each point $g \in U$ there exists a unique function $u \in C^1((-\delta, \delta), U)$ satisfying

$$(1) \qquad \begin{cases} u_t = f(u) & t \in (-\delta, \delta) \\ u(0) = g. \end{cases}$$

(a). Show that if $f \in C^k(U)$ then $u \in C^{k+1}((-\delta, \delta))$.

(b). Modify the proof of (1) to show that δ can be chosen to be uniform on a neighborhood of the initial condition g, that is, with the same conditions as above, if $\overline{B(x, 2\epsilon)} \subset U$, then there exists $\delta > 0$ such that for every $g \in B(x, \epsilon)$ there is a unique function $u \in C^1((-\delta, \delta), U)$ satisfying (1).

5. Let $f \in C(\mathbb{R}^n)$ be radial, that is

$$f(x) \;=\; F(|x|)$$

for some $F \in C(\mathbb{R})$. By integrating an ordinary differential equation in the variable r, find the unique radial function $u \in C^2(\mathbb{R}^n)$ which solves

$$\begin{cases} \Delta u \;=\; f, \\ u(0) = 0, \end{cases}$$

and explain why it is indeed twice differentiable.