MATH 231A PARTIAL DIFFERENTIAL EQUATIONS

Homework 3

Evans Section 2.5, # 4, 5, 6, 9.

Symmetry of Green's function. $U \subset \mathbb{R}^n$ is open and bounded with smooth (i.e. C^∞) boundary. Suppose that for $x, y \in \bar{U}$,

$$G(x,y) \;=\; \Phi(x-y) \;-\; \phi(x,y),$$

where

$$\Phi(x) \;=\; \frac{1}{(n-2)|\partial B(0,1)|\,|x|^{n-2}},$$

and suppose $\phi(x,y)$ is smooth on the set

$$\bar{U} \times \bar{U} \;\setminus\; \{(x,y) : x, y \in \partial U, \quad x = y\},$$

and satisfies

$$\begin{cases} \Delta_y \phi(x,y) = 0, & x \in \bar{U}, \quad y \in U, \\ \phi(x,y) = \Phi(x-y), & x \in \bar{U}, \quad y \in \partial U, \quad y \neq x. \end{cases}$$

By considering the integral

$$\int_{U \setminus (B(x,\epsilon) \cup B(y,\epsilon))} G(x,y) \Delta_y G(z,y)\, dy \;-\; \int_{U \setminus (B(x,\epsilon) \cup B(y,\epsilon))} (\Delta_y G(x,y)) G(z,y)\, dy,$$

show that for $x, z \in U$,

$$G(x,z) = G(z,x).$$