Math 231A, Homework 1, Fall 2005.

Question 1. Let g be a C^2 function with $g(0) = g(\pi) = 0$. Find the unique solution $u(x,t)$ to the heat equation

$$\begin{cases} (\partial_t - \partial_x^2)u = 0, & \\ u(x,0) = g & x \in [0,\pi], \\ u(0,t) = u(\pi,t) = 0, & t \geq 0. \end{cases}$$

with $u, \partial_t u, \partial_x u, \partial_x^2 u$ continuous.

Question 2. Find a function $g \in C^2([0,\pi])$ with $g(0) = g(\pi) = 0$ such that there is no solution $u(x,t)$ to the backward heat equation

$$\begin{cases} (\partial_t + \partial_x^2)u = 0, & \\ u(x,0) = g & x \in [0,\pi], \\ u(0,t) = u(\pi,t) = 0, & t \geq 0. \end{cases}$$

with $u, \partial_t u, \partial_x u, \partial_x^2 u$ continuous. (You can give g as a Fourier series.)

Question 3. Suppose that $g \in C^2([0,\pi])$ with $g(0) = g(\pi) = 0$. Suppose that $u : [0,\pi] \times (-\infty,\infty) \to \mathbb{C}$ is a solution to the Shrödinger equation

$$\begin{cases} (i\partial_t + \partial_x^2)u = 0, & \\ u(x,0) = g & x \in [0,\pi], \\ u(0,t) = u(\pi,t) = 0, & t \geq 0. \end{cases}$$

with $u, \partial_t u, \partial_x u, \partial_x^2 u$ continuous. Write down a series solution for u.

Question 4. Suppose that g is C^2 and h is C^1 on $[a,b]$ with $g(a) = g(b) = 0$. Write down a series for the C^2 solution $u(x,t)$ to the wave equation

$$\begin{cases} (\partial_t^2 - \partial_x^2)u = 0, & \\ u(x,0) = g & x \in [a,b], \\ \partial_t u(x,0) = h & x \in [a,b], \\ u(a,t) = u(b,t) = 0, & t \geq 0. \end{cases}$$

Question 5. Find a function $g \in C^2([0,\pi])$ with $g(0) = g(\pi) = 0$ such that there is no C^2 solution $u(x,t)$ to Laplace's equation

$$\begin{cases} (\partial_t^2 + \partial_x^2)u = 0, & \\ u(x,0) = g & x \in [0,\pi], \\ \partial_t u(x,0) = 0 & x \in [0,\pi], \\ u(0,t) = u(\pi,t) = 0, & t \geq 0. \end{cases}$$

(You can give g as a Fourier series.)

Question 6. Let $g_0, g_1 \in C^2([0,\pi])$ with $g_0(0) = g_0(\pi) = g_1(0) = g_1(\pi) = 0$. Suppose that $u(x,t)$ is a C^2 solution to Laplace's equation

$$\begin{cases} (\partial_t^2 + \partial_x^2)u = 0, & \\ u(x,0) = g_0(x) & x \in [0,\pi], \\ u(x,\pi) = g_1(x) & x \in [0,\pi], \\ u(0,t) = u(\pi,t) = 0, \qquad t \geq 0. & \end{cases}$$

Write down a series solutions for u.