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The art of subtracting infinity

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We begin this lecture by reviewing the concept of the trace of a matrix. Then we move on to study the vibrations of a guitar string of length π. The noise that the string produces contains a base tone and higher overtones. In the idealized mathematical model, the list of frequencies of all these tones is

$$1,\ 2^2,\ 3^2,\ 4^2,\ \dots,$$

and so the wavelengths are

$$1,\ \frac{1}{2^2},\ \frac{1}{3^2},\dots\dots$$

We explain why the *total wavelength*, $1+\frac{1}{2^2}+\frac{1}{3^2}+\dots$ can be computed by integrating the *Green's function* $x(1-x/\pi)$ along the guitar string, giving an answer of $\pi^2/6$.

The trouble starts when we try to replace the guitar string by a sphere and carry out the same process. The resulting formula

$$\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)} \;=\; \int_{S^2} G(x,x)\,dS(x)$$

simply boils down to $\infty=\infty$. However, by subtracting infinity very carefully from each side, we obtain an interesting formula which works for any surface, not just a sphere. Although this mathematics may never be used to help us design musical instruments, it is related to the vortex theory of fluids, and Einstein's relativity.

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