GRE Differential Equation

Summary:
- Separation method
- Technique for homogeneous equation
- Solving an exact equation by variation of parameters
- Solving a non-exact equation by finding an integrating factor or variation of parameter
- Solving a homogeneous higher-order ODE

Since there is no discussion of variation of parameter in the GRE textbook which turns out a very powerful method, let me discuss it here. Using variation of parameter, we can solve an inhomogeneous 1st order ODE

\[
\frac{dy}{dx} + P(x)y = Q(x)
\]

For example

\[
\frac{dy}{dx} = 5x - \frac{3y}{x}
\]  

(1)

First, we consider an homogeneous one

\[
\frac{dy}{dx} = \frac{3y}{x}
\]  

(2)

We have \( y = C \frac{1}{x^3} \) to be a solution of (2)

We assume \( y = C(x) \frac{1}{x^3} \) to be a solution of (1) where \( C(x) \) is a differentiable function. Put the solution to the equation (1). We have

\[
C'(x) \frac{1}{x^3} - \frac{3y}{x} = 5x - \frac{3y}{x} \Rightarrow C'(x) = 5x^4 \Rightarrow C(x) = x^5 + C
\]

Therefore, a solution of equation (1) is

\[
y = \frac{x^5 + C}{x^3}
\]

Q37(96)

Which of the following is the general solution of the differential equation?

\[
\frac{d^3y}{dx^3} - 3 \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - y = 0
\]
Q5 (87)

All functions $f$ defined on the xy-plane such that

$$\frac{\partial f}{\partial x} = 2x + y \quad \text{and} \quad \frac{\partial f}{\partial y} = x + 2y$$

are given by $f(x, y) =$

(A) $x^2 + xy + y^2 + C$ \hspace{1cm} (B) $x^2 - xy + y^2 + C$ \hspace{1cm} (C) $x^2 - xy - y^2 + C$ \hspace{1cm} (D) $x^2 + 2xy + y^2 + C$ \hspace{1cm} (E) $x^2 - 2xy + y^2 + C$

Q40(87)

Let $y = f(x)$ be a solution of the differential equation

$$xdy + (y - xe^x)dx = 0$$

such that $y = 0$ when $x = 1$. What is the value of $f(2)$?

(A) $1/(2e)$ \hspace{1cm} (B) $1/e$ \hspace{1cm} (C) $e^2/2$ \hspace{1cm} (D) $2e$ \hspace{1cm} (E) $2e^2$

Q40(87)

Which of the following indicates the graphs of two functions that satisfy the differential equation

$$\left(\frac{dy}{dx}\right)^2 + 2y\left(\frac{dy}{dx}\right) + y^2 = 0$$

Contact me if you have any question: p1tong@ucsd.edu