Question 20, p.981 on the textbook by Edwards & Penny

How to argue the critical point is minimum?

Since the constraint, \(xyz = 700\), \(x > 0\), \(y > 0\), \(z > 0\) has no boundary.

We should consider, \(x \to +\infty\), \(y \to +\infty\) or \(z \to +\infty\).

I only do the first case.

\[
\lim_{x \to +\infty} f(x, y, z) = \lim_{x \to +\infty} 7xy + 10yz + 10xz \\
= \lim_{x \to +\infty} 7\frac{700}{yz}y + 10\frac{700}{xz}z + 10\frac{700}{yz}z \\
= \lim_{x \to +\infty} \frac{4900}{z} + \frac{7000}{x} + \frac{7000}{y} \\
\geq \lim_{x \to +\infty} \frac{4900}{z} + \frac{7000}{y} \\
\to +\infty \text{ as there are at least } y \text{ or } z \to 0.