solution 7

5. Use chain rule.

\[ w = \ln(x^2 + y^4 + z^2), \quad x = s - t, \quad y = s + t, \quad z = 2\sqrt{st}. \]

\[ \frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}. \]

\[ = \frac{2}{s + t} \]

\[ \frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial t}. \]

\[ = \frac{2}{s + t}. \]

9. \( V = e^{u + v + w}, \quad u = y^2, \quad v = x^2, \quad w = xy. \)

\[ \frac{\partial v}{\partial x} = e^{y^2 + x^2 + xy} (2 + 2x). \]

\[ \frac{\partial v}{\partial y} = e^{y^2 + x^2 + xy} (2 + x). \]

\[ \frac{\partial v}{\partial z} = e^{y^2 + x^2 + xy} (x + y). \]

**Note:** the result should be in terms of the variables which you do the differentiation.
\[ f(x, y, z) = x^3 + y^3 + z^3 - xyz = 0. \]
\[ \frac{\partial f}{\partial z} = 3z^2 - xy. \]

The partial derivative is nonzero whenever \( 3z^2 - xy \neq 0 \).

Hence \( z \) is defined as a function of \( x, y \) except at the points of the curve \( x^3 + y^3 = 2 \left( \frac{xy}{3} \right)^\frac{3}{2} \). At other points of the surface, we can differentiate with respect to \( x \) and \( y \).

\[ 0 = \frac{\partial f}{\partial x} \cdot \frac{dx}{dx} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dx} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dx} \]

\[ \Rightarrow \frac{dz}{dx} = \frac{yz - 3x^2}{3z^2 - xy}. \]

\[ 0 = \frac{\partial f}{\partial x} \cdot \frac{dx}{dy} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dy} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dy} \]

\[ \Rightarrow \frac{dz}{dy} = \frac{xz - 3y^2}{3z^2 - xy}. \]