46. \( w = f(x, y) = 10 + 0.003x^2 - 0.004y^2 \)
\( \nabla f = (0.006, -0.008) \)
At \((40, 30)\), \( \nabla f \big|_{(40,30)} = (0.24, 0.24) \)
\[ |\nabla f| = (0.24^2 + 0.24^2)^{\frac{1}{2}} \]
Hence, \( u = \frac{\nabla f}{|\nabla f|} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \)
Also, directional derivative of \( f \) along \( u \) at \((40, 30)\) =
\[ D_u f = |\nabla f| = 0.3394 \]

16. Let \( f(x, y, z) = z, g(x, y, z) = x^2 + y^2 - 1, h(x, y, z) = 2x + 2y + z - 5 \)
Let \( F(x, y, z) = f(x, y, z) - \lambda g(x, y, z) - \mu h(x, y, z) \)
Consider,
\[ F_x = -2x\lambda - 2\mu = 0 \]
\[ F_y = -2y\lambda - 2\mu = 0 \]
\[ F_z = 1 - \mu = 0 \]
\( \therefore (\lambda, \mu, x, y, z) = (1, \sqrt{2}, 5 + 2\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \) or \((1, -\sqrt{2}, 5 - 2\sqrt{2}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})\)
Since the intersection of the two constraint is a closed curve. i.e. the intersection has no boundary.
Indeed, \( i) \) the intersection is closed and bounded and \( ii) \) \( f \) is continuous.
As a result, \( f(5 + 2\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 5 + 2\sqrt{2} \) is maximum while \( f(5 - 2\sqrt{2}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 5 - 2\sqrt{2} \) is minimum.

18. Let \( f(x, y, z) = x, g(x, y, z) = x + y + z - 12, h(x, y, z) = 4y^2 + 9z^2 - 36 \)
Let \( F(x, y, z) = f(x, y, z) - \lambda g(x, y, z) - \mu h(x, y, z) \)
Consider,
\[ F_x = 1 - \lambda = 0 \]
\[ F_y = -\lambda - 8y\mu = 0 \]
\[ F_z = -\lambda - 18z\mu = 0 \]
\( \therefore (\lambda, \mu, x, y, z) = (\frac{\sqrt{13}}{2}, 12 + \sqrt{13}, -\frac{9}{\sqrt{13}}, -\frac{4}{\sqrt{13}}) \) or \((1, -\frac{\sqrt{13}}{2}, 12 - \sqrt{13}, \frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}})\)
Since the intersection of the two constraint has no boundary.
Indeed, \( i) \) the intersection is closed and bounded and \( ii) \) \( f \) is continuous.
As a result, \( f(12 + \sqrt{13}, -\frac{9}{\sqrt{13}}, -\frac{4}{\sqrt{13}}) = 12 + \sqrt{13} \) is maximum while \( f(12 - \sqrt{13}, \frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}) = 12 - \sqrt{13} \) is minimum.