I did Ex. 2.6 #12
Ex. 2.8 #12
Ex. 3.1 #11, 49, 59, 65
Ex. 3.2 #1 30, 45, 50, 70, 71

Ex. 2.6
Q12 Evaluate using the Squeeze Thm.
\[
\lim_{x \to 0^+} \sqrt{x} e^{\cos(\frac{\pi}{x})}
\]

Pf: \(0 \leq \sqrt{x} e^{\cos(\frac{\pi}{x})} \leq \sqrt{x}\) \(\because |\cos(\frac{\pi}{x})| \leq 1\)

\[
\lim_{x \to 0^+} 0 \leq \lim_{x \to 0^+} \sqrt{x} e^{\cos(\frac{\pi}{x})} \leq \lim_{x \to 0^+} \sqrt{x}
\]

\(0 \leq \lim_{x \to 0^+} \sqrt{x} e^{\cos(\frac{\pi}{x})} \leq 0\)

\[
\lim_{x \to 0^+} \sqrt{x} e^{\cos(\frac{\pi}{x})} = 0
\]

Ex. 2.8
Q12 Prove \(2^x = bx\) has a solution if \(b > 2\) by IVT.

Pf: for \(x = 2\), \(bx = 2b \geq 2 \times 2 = 2^2 = 2^x\)

for \(x = 0\), \(bx = 0 \leq 1 = 2^0 = 2^x\)

\(\therefore\) By IVT, \(\exists \ x_0 \in [0, 2]\) such that
\[2^{x_0} = bx\]
Ex 3.1

Q 11. Determine $f'(a)$ for $a=1, 2, 4, 7$ in the figure 12 in your book.

Pf: $f'(1) = 0$
$f'(2) = 0$
$f'(4) = \frac{2-1}{4-3} = 1$
$f'(7) = 0$ (Why?)

Q 49. Determine the interval along the $x$-axis on which the derivative in figure 15 is positive.

Pf: Please be reminded that the derivative of a function $f$ is positive if and only if $f$ increases strictly on the interval.

i. The derivative is positive in the intervals $(1, 2.5)$ and $(3.5, \infty)$
Q59.
For each graph in Figure 16, determine whether \( f'(1) \) is larger or smaller than the slope of the secant line between \( x=1 \) and \( x=1+h \) for \( h>0 \). Explain.

\[ \text{tangent at } f(1) \]

\[ \text{secant} \]

\[ y = f(x) \]

\[ x \]

(A)

\[ \text{secant} \]

\[ \text{tangent at } f(1) \]

(B)

Proof: For (A), we find that \( f(x) \) increases not large enough so that when you draw a secant between 1 and 1+h, you will observe \( f'(1) > \) the slope of the secant as the tangent line at \( x=1 \) is always above the secant and they have the same y-value at \( x=1 \).

For (B), the tangent line at \( x=1 \) is always below the secant and they have the same y-value at \( x=1 \).

\( \Rightarrow \) slope of the tangent < slope of the secant.

\( \Rightarrow \) \( f'(1) < \) slope of the secant.
The vapor pressure of water at temperature $T$ (in kelvins) is the atmospheric pressure $P$ at which no net evaporation takes place. Use the following table to estimate $P'(T)$ for $T=303, 313, 323, 333, 343$ by computing the $5^\text{th}$ order polynomial given by Eq. (4) with $h=1.0$.

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>293</th>
<th>303</th>
<th>313</th>
<th>323</th>
<th>333</th>
<th>343</th>
<th>353</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (atm)</td>
<td>0.0278</td>
<td>0.0482</td>
<td>0.0808</td>
<td>0.1311</td>
<td>0.2067</td>
<td>0.3173</td>
<td>0.4759</td>
</tr>
</tbody>
</table>

\begin{align*}
P'(303) & \approx \frac{P(313) - P(293)}{20} = 0.00265 \text{ atm/k} \\
P'(313) & \approx \frac{P(323) - P(303)}{20} = 0.004145 \text{ atm/k} \\
P'(323) & \approx \frac{P(333) - P(313)}{20} = 0.006295 \text{ atm/k} \\
P'(333) & \approx \frac{P(343) - P(323)}{20} = 0.00931 \text{ atm/k} \\
P'(343) & \approx \frac{P(353) - P(333)}{20} = 0.013435 \text{ atm/k}
\end{align*}
Ex 3.2

0.4

Compute \( f'(x) \) where \( f(x) = 1 - x^{-1} \)

\[ f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{for} \quad x \neq 0 \]

\[ = \lim_{\Delta x \to 0} \frac{1 - (x + \Delta x)^{-1} - x^{-1}}{\Delta x} \]

\[ = \lim_{\Delta x \to 0} \frac{(1 - \frac{1}{x + \Delta x}) - (1 - \frac{1}{x})}{\Delta x} \]

\[ = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left( \frac{1}{x} - \frac{1}{x + \Delta x} \right) \]

\[ = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left( \frac{x + \Delta x - x}{x(x + \Delta x)} \right) \]

\[ = \lim_{\Delta x \to 0} \frac{1}{x(x + \Delta x)} \]

\[ = \frac{1}{x^2} \]
Q30
Calculate the derivative.
\[ f(x) = 3e^x - x^3 \]

\[ f' = f'(x) = 3e^x - 3x^2 \]

Q45
Assign the labels \( f(x) \), \( g(x) \) and \( h(x) \) to the graph in Figure 15 in such a way that \( f(x) = g(x) \) and \( g(x) = h(x) \).

\[ f' = f(x) \text{ in (A)} \]

\[ g(x) \text{ in (C)} \]

and \[ h(x) \text{ in (B)} \]

Because in (A), \( f \) increase and decrease to back to 0 when its x-value move from 0 to \( a \).
So \( f' \) should be positive and then negative when its x-value moves from 0 to \( a \), that is the picture of (C). Also, \( g'(a) \) in C should be zero which matches the \( h(a) \) in (B). P6
Q50
Find the points on the graph of \( f(x) = 12x - x^3 \) where the tangent line is horizontal.

**Pf.** The tangent line is horizontal means its slope is zero.

It suffices for us to find \( x \) such that \( f'(x) = 0 \)

\[ f'(x) = 0 \]

\[ 12 - 3x^2 = 0 \]

\[ x = \pm \sqrt{4} = \pm 2. \]

\[ \therefore \text{Points are } (2, 16), \ (-2, -16) \]

Q70.
Determine the values of \( x \) at which the function in Figure 21 is (a) discontinuous, and (b) non-differentiable.

**Pf.** \( f \) is not continuous at \( x=1 \).

\( \therefore \) \( f \) is not differentiable at \( x=1 \) (why?)

Also, there is a kink at \( x=2 \) and \( x=3 \), so \( f' \) is not continuous at \( x=2 \) and \( x=3 \).
Q71
Find the points $c$ (if any) such that $f'(c)$ does not exist.

$f(x) = |x-1|.$

pf: for $x > 1$, $f(x) = |x-1| = x-1$

i.e. $f'(x) = 1$

i.e. $f$ is differentiable for $x > 1$

for $x < 1$, $f(x) = |x-1| = -x+1$

i.e. $f'(x) = -1$

i.e. $f$ is differentiable for $x < 1$

for $x = 1$,

\[
\lim_{\Delta x \to 0^+} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{\Delta x}{\Delta x} = 1
\]

\[
\lim_{\Delta x \to 0^-} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{-\Delta x}{\Delta x} = -1
\]

i.e. right-hand limit of $f'(1)$ $\neq$ left-hand limit of $f'(1)$

i.e. $f'(1)$ does not exist.