Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.
1. (6 points) Differentiate the following functions; you need not simplify.

(a) \( f(x) = \frac{x}{2 + \sin(x)} \)

\[
\frac{d}{dx} f(x) = \frac{\dot{2} + \sin(x) - x (2 + \sin(x))}{(2 + \sin(x))^2}
\]

(b) \( g(x) = e^{3x} \cos(4x) \)

\[
g'(x) = 3e^{3x}\cos(4x) + e^{3x}(-4\sin(4x))
\]

\[
= e^{3x}(3\cos(4x) - 4\sin(4x))
\]

(c) \( h(x) = [\cos(x)]^{3x} \)

Let \( H(x) = \log h(x) = x \log(\cos(x)) \)

\[
H'(x) = \frac{d}{dx} x \log(\cos(x)) = \log(\cos(x)) + x \cdot \frac{1}{\cos(x)} (-\sin(x))
\]

\[
= \log(\cos(x)) - x \tan(x)
\]

Also \( (\log h(x))' = \frac{h'(x)}{h(x)} \)

\[
h'(x) = \left( \frac{h(x)}{\log(\cos(x)) - x \tan(x)} \right)
\]

\[
= \left[ \cos(x) \right]^{3x} \left( \log(\cos(x)) - x \tan(x) \right)
\]
2. (8 points) Let \( f(x) = x + \frac{1}{x - 3} \). The first and second derivatives of \( f \) are given by:

\[
\begin{align*}
 f'(x) &= 1 - \frac{1}{(x-3)^2} \\
 f''(x) &= \frac{2}{(x-3)^3}.
\end{align*}
\]

(a) Find the interval(s) on which \( f \) is increasing and the interval(s) on which \( f \) is decreasing.

By the First Derivative,

\[
f(2) \text{ is local max where } f'(2) = 1
\]

\[
f(4) \text{ is local min where } f'(4) = 5
\]

(b) Find the local maximum and local minimum values of \( f \).

(c) Find the interval(s) on which the graph of \( f \) is concave upward and the interval(s) on which the graph of \( f \) is concave downward.

\[
f''(x) = 0
\]

\[
\frac{2}{(x-3)^3} = 0
\]

no soln

And

(d) Find the vertical asymptote(s), if any.

\[
\left| \begin{array}{c}
\text{for } x \rightarrow 3 \\
\frac{1}{x-3} \rightarrow +\infty
\end{array} \right|
\]

\( x = 3 \) is a vertical asymptote of \( f \).
3. (6 points) Use the fact that $27^{\frac{1}{3}} = 3$ to find a linear approximation for $(27.03)^{\frac{1}{3}}$.

$$y = x^{\frac{1}{3}}$$

$$y' = \frac{1}{3} x^{-\frac{2}{3}}$$

$$27.03 = (27 + 0.03)^{\frac{1}{3}}$$

$$\approx (27)^{\frac{1}{3}} + \frac{1}{3} (27)^{\frac{2}{3}} \cdot 0.03$$

$$= 3 + \frac{1}{3} \left( \frac{1}{9} \right) \frac{3}{100}$$

$$= 3 + \frac{1}{300}$$
4. (6 points) Randall Cohn has a pool with the shape of an inverted cone which is 5 meters deep with a radius of 5 meters at the top (base). Randall fills the pool with his garden hose at a rate of 0.1 cubic meters per minute. At what rate is the water depth increasing when the depth is 3 meters? (Note: The volume of a cone of height $h$ and radius $r$ is given by $V = \frac{1}{3} \pi r^2 h$.)

\[
V = \frac{1}{3} \pi r^2 h
\]

\[
\frac{r'}{5} = \frac{h'}{5}
\]

\[
h = r'
\]

\[
V(t) = \frac{1}{3} \pi h^3
\]

\[
\frac{dV}{dt} = \frac{1}{3} 3\pi h^2 \frac{dh}{dt}
\]

\[
= \pi h^2 \frac{dh}{dt}
\]

\[
0.1 = \pi (3)^2 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{0.1}{9\pi} = \frac{1}{90\pi}
\]
5. (6 points) Find the point(s) on the ellipse \( x^2 + xy + y^2 = 12 \) at which the corresponding tangent line is horizontal.

\[
x^2 + xy + y^2 = 12 \quad \text{by (1)}
\]

\[
2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \quad \text{by (2)}
\]

Now, if \((x, y)\) is the point on the ellipse such that the corresponding tangent line is horizontal, it means \( \frac{dy}{dx} = 0 \)

\[
\begin{align*}
2x + y &= 0 \quad \text{by (2)} \\
x^2 + xy + y^2 &= 12 \quad \text{by (1)}
\end{align*}
\]

By substitution method, we set \( y = -2x \) and substitute it to the equation \( x^2 + xy + y^2 = 12 \)

\[
x^2 + x(-2x) + (-2x)^2 = 12
\]

\[
x^2 - 2x^2 + 4x^2 = 12
\]

\[
3x^2 = 12
\]

\[
x^2 = 4
\]

\[
x = \pm 2 \quad \text{or} \quad -2
\]

\[
\therefore (2, -4) \text{ and } (-2, 4) \text{ are the answer.}
\]