I did Ex/4.1 Q16, Q18, Q19, Q24, Q32
Ex/4.2 Q18, Q24, Q26, Q28
Ex/4.3 Q49

Ex. 14.1
Q16 Describe the domain and range of function.
\[ g(r,s) = \cos^{-1}(r,s) \]

\[ \text{pf: As } \cos^{-1} \text{ has range from } -1 \text{ to } 1 \]
\[ \text{we need the domain is } \]
\[ -1 \leq rs \leq 1, \text{ or } |rs| \leq 1 \]
\[ \text{Ans: } D = \{ (r,s) : |rs| \leq 1 \} \]
\[ \text{For the range of } \cos^{-1}, \text{ it is } [0, \pi]. \]

Q18.
Match each of graphs (A) and (B) with one of the following functions
(i) \[ f(x, y) = \cos x \cos y \]
(ii) \[ g(x, y) = \cos(x^2+y^2) \]

\[ \text{pf: The level curve of } g \text{ (A)} \]
\[ \text{is } \cos(x^2+y^2) \text{, it is a series of circles.} \]
\[ \text{If } x^2+y^2 = C \text{ and } B \text{ has level curves circles.} \]
Q19.

\[ (e) \text{ has level curves circles and those curves, fixed } c, \{ (x, y, z) : f(x, y) = c, z = c \} \]
\[ \text{are below } xy\text{-plane} \]

\[ (c) \text{ has level curves ellipses and those curves, fixed } c, \{ (x, y, z) : f(x, y) = c, z = c \} \]
\[ \text{are below } xy\text{-plane} \]

\[ (f) \text{ has level curves are circles but some of those curves, fixed } c, \{ (x, y, z) : f(x, y) = c, z = c \} \]
\[ \text{are above } xy\text{-planes} \]

\[ (f) \leftrightarrow F \]
(a) has the level curves like \( \square \),
and these curves, fixed \( c \), \( f(x, y, z) = f(x, y) = c \),
\( z = c \), are above \( xy \)-plane.

\( \therefore (a) \leftrightarrow D \)

(b) has the level curves like \( x - y = c \).

\( \therefore (b) \leftrightarrow C \)

Finally, \( (d) \leftrightarrow (B) \)

Q 24
Sketch the graphs and describe the vertical and horizontal traces.

\( f(x, y) = y^2 \)

\[ \text{This is the graph } f(x, y) = y^2. \]
The vertical trace

\[ \begin{align*}
  \text{horizontal line} & \quad y = c \quad (c > 0) \\
  \text{NO line below} & \quad x = \text{axis} \\
  \text{set} \quad x = \text{constant } c
\end{align*} \]

The horizontal trace

\[ \begin{align*}
  \text{as} \quad f(x, y) = c \quad (c > 0) & \quad \Rightarrow y^2 = c \\
  \text{two horizontal lines} & \quad \Rightarrow y = \sqrt{c} \text{ or } y = -\sqrt{c}.
\end{align*} \]
Q32. Draw a contour map of \( f(x, y) = xy \). (six contour curves)

pf:

\[ xy = \alpha, \, \pm 1, \, \pm 2, \, \ldots \]

\[ + \quad \text{as } xy = 0 \]
Ex. 14.2

Q18.

Evaluate \( \lim_{(x,y) \to (0,0)} (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) \)

**Pf.** As \(-1 \leq \sin x \leq 1\)

\[ -\tan x \leq (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) \leq \tan x \]

as \( x \to 0^+ \), \( \tan x \to 0 \)

\[ 0 \leq \lim_{(x,y) \to (0,0)} (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) \leq 0 \]

\[ \lim_{(x,y) \to (0^+,0)} (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) = 0 \]

By **Squeeze's**

If \( x \leq 0 \)

\[ -\tan x \geq (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) \geq \tan x \]

as \( x \to 0^- \), \( \tan x \to 0 \).

\[ 0 \geq (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) \geq 0 \]

\[ \lim_{(x,y) \to (0^-0)} (\tan x)(\sin \left( \frac{1}{|x|+|y|} \right)) = 0 \]

By **Squeeze's**

\[ \text{R}6 \]
Q24 Determine if \( \lim_{{(x,y) \to (0,0)}} \frac{xy}{\sqrt{x^2 + y^2}} \) exists.

And what is it if it exists.

If:

\[
0 \leq \left| \frac{xy}{\sqrt{x^2 + y^2}} \right| = |x| \left| \frac{y}{\sqrt{x^2 + y^2}} \right| \leq 1|\text{x}|
\]

As \((x,y) \to (0,0)\), \(1|\text{x}| \to 0\)

\[
0 \leq \lim_{{(x,y) \to (0,0)}} \left| \frac{xy}{\sqrt{x^2 + y^2}} \right| \leq 0
\]

\[
\therefore \lim_{{(x,y) \to (0,0)}} \frac{xy}{\sqrt{x^2 + y^2}} = 0 \quad \text{by Squeeze Thm.}
\]

Q26

Determine if \(\lim_{{(x,y) \to (0,0)}} \frac{1|\text{x}|}{1|\text{x}|+1|\text{y}|} \) exists.

PF: Let \(x = 0, \ y \to 0\)

\[
\lim_{{y \to 0}} \frac{1|\text{x}|}{1|\text{x}|+1|\text{y}|} = 0
\]

Let \(x = t, \ y = t, \ t \to 0^*\).
\[
\lim_{{t \to 0}} \frac{101}{101 + 101} = \lim_{{t \to 0}} \frac{101}{2 \cdot 101} = \frac{1}{2}.
\]

As both limits in different paths are different, \(\lim_{{(x,y) \to (0,0)}} \frac{101}{101 + 101}\) does not exist.

Q228

Determine if \(\lim_{{(x,y) \to (2,1)}} e^{x^2 - y^2}\) exists and what is it?

\[\text{Pf. }\quad \text{As } e^{x^2 - y^2} \text{ is continuous at } (2,1)\]

\[\therefore \lim_{{(x,y) \to (2,1)}} e^{x^2 - y^2} = e^{2^2 - 1^2} = e^3\]