#3.1 THE CONVERSE IS NOT TRUE. FOR A COUNTEREXAMPLE, CONSIDER THE SEQUENCE

$$s_n : (-1)^n.$$

THEN $|s_n| = 1 \ \forall n$, AND SO s_n CLEARLY CONVERGES TO 1. HOWEVER, IT IS OBVIOUS THAT s_n DOES NOT CONVERGE TO ANY NUMBER. (CHECK THIS!)

PROF. ROTHSCHILD'S MIDTERM

~~[illegible]~~

4b) FALSE. CONSIDER THE SEQUENCE

$$p_n = \begin{cases} \frac{1}{n} & n = 2k,\ k \in \mathbb{N} \\ 1 + \frac{1}{n} & n = 2k+1,\ k \in \mathbb{N} \end{cases}$$

THE FIRST SEVERAL TERMS OF p_n ARE

$$\left\{2, \frac{1}{2}, \frac{4}{3}, \frac{1}{4}, \frac{6}{5}, \frac{1}{6}, \frac{8}{7}, \ldots\right\}$$

LET $p = 0$. THEN EVERY NEIGHBORHOOD OF 0 ~~WITH~~ RADIUS LESS THAN 1 WILL CONTAIN INFINITELY MANY ~~[illegible]~~ p_n BUT WILL NOT CONTAIN ALL BUT FINITELY MANY p_n, SINCE INFINITELY MANY p_n ACCUMULATE NEAR 1.

d) TRUE. SINCE $\left|\frac{a_n}{n^2}\right| \le \frac{n^2+n+1}{n^2} = 1 + \frac{1}{n} + \frac{1}{n^2} \le 3$,

$-3 \le \frac{a_n}{n^2} \le 3$. HENCE $\left\{\frac{a_n}{n^2}\right\}$ IS BOUNDED AND THEREFORE CONTAINS A CONVERGENT SUBSEQUENCE. THE SUBSEQUENTIAL LIMIT α IS A LIMIT POINT OF $\left\{\frac{a_n}{n^2}\right\}$ (WHY?)

FIRST PRACTICE MIDTERM

3d) THIS IS FALSE. ONE CAN THINK OF THE SEQUENCE x_n AS POSSIBLY MOVING TO INFINITY "TOO FAST" FOR THE SEQUENCE y_n^n. TAKE AS A COUNTEREXAMPLE:

$$x_n := n^n.$$

$$y_n := \frac{1}{n}.$$

CLEARLY, $x_n \xrightarrow{n\to\infty} \infty$, $y_n \xrightarrow{n\to\infty} 0$, YET

$$x_n y_n^n = \frac{n^n}{n^n} \equiv 1. \quad \text{THUS } x_n y_n^n \xrightarrow{n\to\infty} 1 \neq 0.$$

SECOND PRACTICE MIDTERM

3b) FALSE. ONE OF THE SEQUENCES ALREADY MENTIONED HERE MAKES A GOOD COUNTEREXAMPLE. WHICH IS IT, AND WHY?

3c) TRUE. THIS IS PART OF THE STATEMENT OF THM 3.4a. YOU SHOULD KNOW THIS PROOF!

3g) FALSE. LET $b_n = n^n$. THEN

$$(b_n)^{1/n^2} = n^{1/n} \xrightarrow{n\to\infty} 1 \quad (\text{THM } 3.20c)$$

3h) FALSE. LET $w_n \equiv 1$, $z_n = \frac{1}{n}$. THEN $\frac{w_n}{z_n} = n$, WHICH DOES NOT CONVERGE.