# Homework 6 Solutions

MTH 140B

**1.** By considering its power series expansion, one can easily verify that $e^{\bar{z}} = \overline{e^z}$. Hence, it follows that

$$\sin^2(t) + \cos^2(t) = |\cos(t) + i\sin(t)|^2 = |e^{it}|^2 = e^{it}(\overline{e^{it}}) = e^{it}e^{-it} = e^0 = 1.$$

Also, one has that $\frac{d}{dt}(e^{it}) = ie^{it} = i\cos(t) - \sin(t)$. The equations for the derivatives of $\sin(t)$ and $\cos(t)$ then follow by identifying real and imaginary parts. The remaining identities follow immediately from expanding the identity $e^{a+b} = (e^a)(e^b)$.

**2.** **(a)** The reader is invited to consult pg. 182 of Rudin for further information.

**(b)** As the real part of $e^{it}$, it is clear that $\cos(t)$ is continuous. Hence, the inverse image of $0$, a closed set, must be closed. Thus the inverse image of $0$ under $\cos(t)$ must contains its infimum, which proves the claim.

**(c)** The reader is invited to consult pg. 183 of Rudin for further information.