One direction is clear. If \( f_n \rightarrow f \) in \( H(G) \), then \( f_n \rightarrow f \) \( L^2 \) is compact.

Suppose \( f_n \rightarrow f \) \( L^2 \) rectifiable curve in \( G \). To show that \( f_n \rightarrow f \) in \( H(G) \), pick any \( \overline{B}(r, 2r) \subseteq G \). Pick any \( z \in \overline{B}(r, 2r) \) where \( r < R \). By Cauchy integral formula,

\[
|f_n(z) - f(z)| = \left| \frac{1}{2\pi i} \int_{\partial \Omega} \frac{f_n(w) - f(w)}{w-z} \, dw \right| \leq \frac{1}{2\pi} \cdot 2\pi r \cdot \sup_{w \in \partial \Omega} |f_n - f| \xrightarrow{r \to 0} 0 \quad \text{as} \ n \to \infty
\]

b/c \( f_n \Rightarrow f \) by assumption. And more importantly, this is independent of \( z \in \overline{B}(r, 2r) \).

We've proven \( f_n \Rightarrow f \) \( L^2 \) in \( G \), so \( f_n \) converge to \( f \) locally uniformly, hence \( f_n \rightharpoonup f \)

Suppose \( f_n \rightharpoonup f \) in \( H(G) \). Then \( \exists k \in \mathbb{C}^+ \) s.t. \( f_n \neq f \), i.e., \( \|f_n - f\| \neq 0 \).

\[\exists \varepsilon > 0 \quad \text{such that} \quad \|f_n - f\| > \varepsilon \quad \text{for all} \ n \in \mathbb{N}\]

On the other hand, \( f_n \) locally \( \Rightarrow \) normal \( \Rightarrow \) Every sequence has "most!" convergent subsequence.

In particular, \( f_n \) \( \subseteq f_n \) has convergent subsequence, say \( f_n \rightharpoonup f \).

Put \( f_n \rightharpoonup g \) in \( H(G) \). By assumption, \( f(z) = g(z) \) \( \forall z \in A \) and then by identity principle, \( f = g \). \( \Rightarrow f_n \rightharpoonup f \) in \( H(G) \), hence \( f_n \rightharpoonup f \).

But this contradicts that \( \|f_n - f\| > \varepsilon \) \( \forall n \). \( \therefore f_n \rightharpoonup f \) in \( H(G) \)
Suppose \( f_n \to f \) in \( H(G) \) and \( f_n \) is 1-1 \( \forall n \). To prove by contradiction,

if \( f \) is neither 1-1 nor constant. Then \( \exists a \neq b \in G \) s.t. \( f(a) = f(b) = x \)

Clearly, \( f_n(a) \to f(a) \). Then \( f_n \) is still 1-1 and \( g \to 0 \).

Let's separate \( a, b \) which are zeros so that \( ab \) is the only zero in \( G \).

Then, by Hurwitz's theorem, \( \exists N \) s.t.

\[
\# \text{ of Zero } (g_n, B(a, r)) = \# \text{ of Zero } (g, B(a, r)) \geq 1
\]

\[
\# \text{ of Zero } (g_n, B(b, r)) = \# \text{ of Zero } (g, B(b, r)) \geq 1
\]

\[
\exists a' \in B(a, r) \text{ s.t. } g_n(a') = 0 \\
\exists b' \in B(b, r) \text{ s.t. } g_n(b') = 0
\]

\[
f_n(a') = f_n(b') \text{ but } a' \neq b' \text{ by separation. This contradicts that } f_n \text{ is 1-1.}
\]