(a) Given analytic continuation along a chain of discs \( \ell(B, R)^{t+1} \) for \( \frac{k}{n} < t < \frac{k+1}{n} \), i.e., piecewise linear curve.

- \( R_t := \max \| y_i - (n+1)t \| d_i; \) \( R_t^+ = (n+1)t \).
- \( B_t := B(\mathcal{C}_t R_t) \).

Check that \( B_t \subset D_{i+1} \) or \( B_t \subset D_i \). We can define \( y_t := \left\{ \begin{array}{ll} i & \text{if } B_t \subset D_i; \\
 & \text{if } B_t \subset D_{i+1}. \end{array} \right. \)

This is well-defined b/c \( f_i f_{i+1} \) agrees on \( D_{i+1} \cap D_i \).

\( \Rightarrow \) \( f(x, B_0) \) satisfies desired condition. (Check)

(b) Given an analytic continuation along a path \( \gamma : [0,1] \rightarrow C \) \( \ell(B, R)^{t+1} \).

Let's say \( \ell(B, R)^{t+1} \) is compatible with some analytic continuation along a chain of discs if we can solve the given problem. We will use connectedness of \([0,1]\) as usual.

Define \( T := \left\{ s \in [0,1] \mid \ell(B, R)^{t+1} \text{ is compatible with some } \ldots \right\} \). Claim: \( T = [0,1] \).

- \( 0 \in T \).
- \( T \) is open: \( s \in T \Rightarrow \exists \text{ analytic continuation along a chain of discs } \ell(B, R)^{t+1} \text{ is compatible with some } \ldots \).

Since \( \gamma(0) \) is U.D., \( \exists \gamma > 0 \) s.t. \( s - \frac{1}{2} < \frac{1}{2} \Rightarrow \gamma(0, s) \subset U.D. \).

\( = (s - \frac{1}{2}, s + \frac{1}{2}) \subset T \), hence open.

- \( T \) is closed: \( s \notin T \). Consider \( \gamma(s, r) \in B_0 \). Supp \( \exists \gamma \in \ell(B(\gamma(0), T). \)

Then by simply adding \( f_{\gamma(0)} B(\gamma(0), r) \) at the end of the chain of discs, we can show that \( s \in T \).

This implies \( t(B(\gamma(0), T) \subset T \Rightarrow T \text{ closed.} \)

\( \Rightarrow T = [0,1] \). \( \therefore \) We are done. \( \square \)
Let \((f, D)\) admits unrestricted continuation to \(G \supset D\).

Supp \(v_0 \subseteq \mathcal{F}_1\), i.e., \(\exists \Gamma : \mathcal{E}_1 \times \mathcal{E}_1 \to G\).

By assumption, \(v_n = \Gamma (-n, n)\) admits analytic continuation of \((f, D)\), say \(v(f(e_n), D(e_n))_{e \in \mathcal{E}_1}\).

Exercise 3 says \(R(e_n) = 0\) or \(R(e_n) : \mathcal{E}_1 \times \mathcal{E}_1 \to \mathbb{R}^+\) continuous.

If \(R(e_n) = 0\), then \((f, D)\) is actually entire function, and so \([f]_b = [g]_b\).

The other case, by compactness, \(\exists \epsilon \geq 0\) such that \(\epsilon < \epsilon < \frac{1}{2} \min |R(e_n)|\).

By uniform continuity of \(\Gamma\), \(\exists \delta > 0\) such that \(\|e_n - e_m\| < \delta \Rightarrow \|\Gamma(e_n) - \Gamma(e_m)\| < \epsilon\).

Pick any \(|v_n(e_t) - v_m(e_t)| < \epsilon\). Then \([v_n]_b = [v_m]_b\).

Pick \(N \in \mathcal{N}\) such that \(N > \frac{1}{2}\). Then \([f_n]_b = [f_{n+1}]_b = [f_{n+2}]_b = \ldots = [f_{\infty}]_b\) by induction.

Thus \([f]_b = [g]_b\).