\[ (a) \ G \text{ is simply connected, hence by Corollary 4.18, } G \text{ is Dirichlet. Also } G \text{ is bounded, so by } \\
\text{Theorem 5.2 there is a Green's function } g_0(z) \text{ on } G \text{ with singularity } \alpha. \\
\begin{align*}
|f(z)| &= |e^{iz} - \alpha - 12 - \alpha| \\
&= e^{-z-\alpha - 12 - \alpha} \quad (z \neq 0) \\
&= e^{-z-\alpha} \\
&= e^{-g_0(z)}
\end{align*}
\]

So \[ \lim_{z \to w} f(z) = \lim_{z \to w} e^{-g_0(z)} = 1, \text{ for any } w \in \partial G. \]

For \( 0 < r < 1 \), let \( G_r = \{ z : |f(z)| \leq r \} \). \( G_r \) is compact, hence \( f' \) has at most finitely many zeros on \( G_r \). This implies that \( f' \) has at most countably many zeros on \( G \). So there exists \( r \) arbitrarily close to 1, such that \( f(z) \neq 0 \) for all \( z \in G_r \). For such \( r \), \( f \) is locally diffeomorphic for any \( B(z; 8(z)), z \in G_r \).

Then using the fact that \( G_r \) is compact, we can show that each \( z \in G_r \) will induce a simple closed curve in \( G_r \) that contains \( z \). The number of such curves must be finite, otherwise we can show that \( f \) is a constant on \( G \).

Now for \( 0 < r < 1 \), let \( G_r = \{ z : |f(z)| \leq r \} \). \( G_r \) is compact, hence \( f' \) has at most finitely many zeros on \( G_r \). So by Rouché's Theorem, \( f \) and \( g \) has the same number of zeros in \( G_r \). Notice that \( f(z) = 0 \) iff \( z = 0 \). So \( f(z) = w \) has unique solution. This means \( f \) is injective on \( G_r \). Since \( r \) is arbitrarily close to 1, we have \( f \) is injective on \( G \), so \( f(z) \neq 0 \) on \( G \). This means that the argument for closed curves work for any \( 0 < r < 1 \). Also we have \( f(G) = D \). If \( f'(a) > 0 \) doesn't hold, consider \( f' \frac{g_0}{f'(a)} \).

(b) Suppose \( f(a) = f(b) \), then \( a = e^{f(a)} = e^{f(b)} = b \), so \( f \) is one-one.

If there exists \( z \in G \), such that \( f'(z) = \alpha + 2\pi i \), then \( z = e^{f(z)} = e^{\alpha} = z \), but \( (\alpha) = \alpha \), contradiction.

By Open Mapping Theorem, there exists \( \epsilon > 0 \) such that \( B(\alpha; \epsilon) \subseteq f(G) \), then by above argument, for any \( w \in B(\alpha; \epsilon) \), we have \( w \in f(G) \), hence \( B(\alpha + 2\pi i; \epsilon) \cap f(G) = \emptyset \). So \( |g(z)| \leq \frac{1}{\epsilon} \).

On the other hand, \( g'(z) = \frac{1}{f'(z)} \neq 0 \), so \( g \) is conformal map onto a bounded simply connected region.

(c) WLOG, we can say \( \alpha = 0 \), \( G \), otherwise we take a \( e^{\alpha} G \), then consider \( f(z + \alpha) \). Now we can use (b) to find a map \( g \) such that \( f(G) \) is bounded simply connected. Then use part (a) to find \( f \) on \( f(G) \), \( f \circ g \) will satisfy all the requirement.