MATH 220A
Midterm Exam, November 8, 2019

Instructions: 1 hour. You may use without proof results proved in Conway up to and including Chapter III. When using a result from the text, be sure to explicitly verify all hypotheses in it. Present your solutions clearly, with appropriate detail.

1. (15+15=30p) Compute the radii of convergence of the following power series:

(a) \( \sum_{n=0}^{\infty} a^n z^{n^2} \)
(b) \( \sum_{n=1}^{\infty} n^2 a^n z^{n^2-1} \),

where \( a \in \mathbb{C} \) and \( a \neq 0 \).
2. Let $S(z)$ be a Möbius transformations such that $S$ maps lines in $\mathbb{C}$ to lines. Determine all such $S(z)$ that also have $k$ fixed points in $\mathbb{C}$, where

(a) $k = 2$  
(b) $k = 1$  
(c) $k = 0$. 
3. (30p) Let \((X, d), (\Omega, \rho)\) be metric spaces, and \(G \subset X, \Delta \subset \Omega\) open subsets. A map \(f : G \rightarrow \Delta\) is called proper if \(f^{-1}(K) \subset G\) is compact for every compact \(K \subset \Delta\). (The empty set is compact.) Suppose that \(f : \overline{G} \rightarrow \overline{\Delta}\) is continuous and and its restriction to \(G\) is a proper map \(G \rightarrow \Delta\). Show that \(f(\partial G) \subset \partial \Delta\).