Recall: FEP-homotopy, $\gamma_{FEP}$.

- Independence of Path Thm from Lecture 23 notes.
- Counting zeros from Lecture 23 notes.

Open Mapping Thm. Let $G \subseteq \mathbb{C}$ be a region, and $f$ analytic and nonconstant in $G$. Then, $f(U)$ is open for every open $U \subseteq G$.

**Pf:** Pick $a \in f(U)$, and $a \in U \Rightarrow f(a) = a$. Since $f$ nonconstant, $f(z) - a$ has a zero of finite multiplicity at $a$. By previous Thm, if $B(a, \delta) \subseteq U$ and $B(a, \epsilon) \cap f(U) = \emptyset$ has $m$ simple roots in $B(a, \delta)$ for each $\epsilon < \delta$.

In particular, $B(a, \delta) \subseteq f(U) \Rightarrow f(U)$ open. \[ \Box \]

Corollary 1. If $f$ is analytic and 1:1 in $G$, then $\Omega = f(G)$ is open and $f^{-1}: \Omega \to G$ analytic.

**Pf:** Previous Prop $\Rightarrow$ if $f^{-1}$ is cont. and $f \neq 0$ in $G$, then $f^{-1}$ is analytic. w/ $(f^{-1})'(z) = \frac{1}{f'(f^{-1}(z))}$.

Well, OM Thm $\Rightarrow f^{-1}$ is cont. Moreover, previous Thm shows that if $f(a) = 0$, then $f \neq 0$ at least, 2:1 near $a$. Since $f$ is globally 1:1 in $G$, by assumption, we conclude $f \neq 0$ in $G$. Thus, $f^{-1}$ is analytic, and $(f^{-1})'(z) = \frac{1}{f'(f^{-1}(z))}$. 