• Finish pf of Thm 6 + continuity principle from Lecture 16 notes.

We can now show that pseudouniqueness is a local property near $\Omega$.

**Thm 1.** Let $\Omega \subseteq \mathbb{C}^n$. If $\forall z \in \Omega$ there exists an open nbhd $U_z \subseteq \mathbb{C}^n$ s.t. $\Omega \cap U_z$ is $\psi$-convex, then $\Omega$ is $\psi$-convex.

**Pf.** For each $z \in \Omega$ $\exists U'_z \subseteq U_z$ s.t. $\delta(z, \mathbb{C}^n - \Omega) = \delta(z, \mathbb{C}^n - (\Omega \cap U'_z))$, for $z \in \Omega \cap U'_z$.

By assumption, $-\log \delta(z, \mathbb{C}^n - (\Omega \cap U'_z))$ is PSH in $\Omega \cap U'_z$ $\Rightarrow$

$u(z) := -\log \delta(z, \mathbb{C}^n - \Omega)$ is PSH in $\Omega \cap U'_z$ $\Rightarrow \exists$ closed $F \subseteq \Omega$

s.t. $u$ is PSH in $\Omega \cap F$. Let $\varphi$ be a convex fn on $|z|^2$ s.t. $\varphi(|z|^2) > u(z)$ on $F$. (Consider $M(r) = \sup_{z \in F \cap u^{-1}(r)} u(z)$, then $M(r) \uparrow$.)

Now let $\varphi(r)$ be convex majorant. Since $\varphi \in \text{PSH}(\mathbb{C}^n)$, $\varphi > u$ on open nbhd of $F$, the fn $v = \max(u, \varphi)$ is PSH in $\Omega$.

Clearly, $\Omega_c := \{z \in \Omega: v(z) < 0\}$ is precompact in $\Omega$ (i.e. satisfies (ii) in Thm 6) $\Rightarrow \Omega$ is $\psi$-convex. $\square$