

Name: \_\_\_\_\_ PID: \_\_\_\_\_

TA: \_\_\_\_\_ Sec. No: \_\_\_\_\_ Sec. Time: \_\_\_\_\_

**Math 10B.**  
**Final Examination**  
**March 21, 2007**

*Turn off and put away your cell phone.*

*You may use any type of calculator, but no other electronic devices during this exam.*

*You may use one page of notes, but no books or other assistance during this exam.*

*Read each question carefully, and answer each question completely.*

*Show all of your work; no credit will be given for unsupported answers.*

*Write your solutions clearly and legibly; no credit will be given for illegible solutions.*

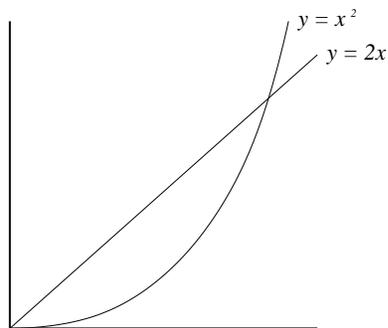
*If any question is not clear, ask for clarification.*

1. (6 points) Using the table below, estimate  $\int_0^6 g(t) dt$  using *LEFT*(2), *RIGHT*(2), and *TRAP*(2).

$t$	0	2	4	6
$g(t)$	1	4	8	17

#	Points	Score
1	6	
2	6	
3	6	
4	8	
5	9	
6	6	
7	8	
8	9	
$\Sigma$	58	

2. Let  $\mathcal{R}$  be the region bounded by the curves  $y = x^2$  and  $y = 2x$ .



(a) (3 points) Find the area of the region  $\mathcal{R}$ .

(b) (3 points) Find the volume of the solid resulting from rotating the region  $\mathcal{R}$  about the  $x$ -axis.

3. Consider the initial value problem

$$\begin{cases} \frac{dV}{dt} = -3(V - 1)^2 \\ V(0) = 1 \end{cases}$$

(a) (4 points) Solve the initial value problem for  $V = V(t)$ .

(b) (2 points) Find  $V(4)$ .

4. On Planet  $\Phi$ , the acceleration of gravity is  $-20 \text{ ft/sec}^2$ . A rock is thrown vertically upward with a velocity of  $20 \text{ ft/sec}$  from the top of a  $30 \text{ ft}$  tower on Planet  $\Phi$ .

(a) (3 points) How long does it take the rock to reach its maximum height?

(b) (3 points) How long does it take the rock to hit the ground?

(c) (2 points) What is the speed at which the rock hits the ground?

5. Evaluate the following integrals.

(a) (3 points)  $\int_0^1 x e^{-6x} dx$

(b) (3 points)  $\int_0^1 x e^{-6x^2} dx$

(c) (3 points)  $\int \frac{10x}{(x-3)(x+7)} dx$

6. Let  $G(x) = \int_0^x 4t^3 \sqrt{1+t^6} dt$ .

(a) (2 points) Compute  $G(0)$ .

(b) (2 points) Compute  $G'(1)$ .

(c) (2 points) Without computing the integral, explain why  $G(1) - G(-1) = 0$ .

7. For each of the following improper integrals, determine whether or not it converges; if it converges, find its value.

(a) (4 points)  $\int_2^6 \frac{1}{(x-2)^{2/3}} dx$

(b) (4 points)  $\int_1^{\infty} \frac{2}{x(2+\ln(x))} dx$

8. Mrs. Smith won 10,000,000 dollars in the lottery in 1997 and put the money in a bank account earning 8 percent interest per year, compounded continuously. Since then, Mrs. Smith has been spending money in this account continuously at a constant rate of  $S$  dollars per year.

(a) (2 points) Write the differential equation and initial condition that describes the balance  $B(t)$  in Mrs. Smith's bank account.

(b) (4 points) Solve the differential equation for  $B = B(t)$  that satisfies the initial condition.

(c) (3 points) Now, ten years after she won the money, Mrs. Smith notices that all the money in her account is gone. How much money did Mrs. Smith spend per year? That is, what is the value of  $S$ ?