

Midterm Exam II

Math 10B
10/15/08

Name: _____
Section: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Give your answers in exact form, for instance:

$$\frac{1}{2}(e^\pi + e^{2\pi}).$$

as opposed to 279.31617.

- A single $8\frac{1}{2} \times 11$ sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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1. (20 points)

a. (10 pts) Find the partial fractions decomposition of

$$\frac{x^2 - 5x - 19}{(x + 2)^2(x - 3)}.$$

Note: you need not integrate.

b. (10 pts) The partial fractions decomposition of $\frac{5x^2+10}{(x-1)(x^2+4)}$ is

$$\frac{5x^2 + 10}{(x - 1)(x^2 + 4)} = \frac{3}{x - 1} + \frac{2x + 2}{x^2 + 4}.$$

Find $\int \frac{5x^2+10}{(x-1)(x^2+4)} dx$.

2. (20 points) Find the following definite or indefinite integrals.

Show all steps: That is, don't just appeal to a formula.

a. (10 pts)

$$\int \sin(2x)e^{3x} dx.$$

b. (10 pts)

$$\int_0^5 \frac{dx}{(5-x^2)^{5/2}}.$$

3. (20 points) For this problem $LHS(k)$, $RHS(k)$, $MIDPOINT(k)$, and $TRAP(k)$ refer to the left hand sum, right hand sum, midpoint rule and trapezoid rule with k boxes, respectively.

a. (5 pts) Write a midpoint approximation with 3 boxes for

$$\int_0^{3\pi} \sin^6(x) dx.$$

b. (8 pts) Suppose the various rules are used to approximate $\int_0^{10} e^{-\pi x} dx$ with k boxes. Arrange the following in *increasing* order: $LHS(k)$, $RHS(k)$, $MIDPOINT(k)$, $TRAP(k)$ and $\int_0^{10} e^{-\pi x} dx$. How do you know this?

c. (7 pts) Suppose the $f(x)$ is a line, and the midpoint and trapezoid rules are used to approximate $\int_0^1 f(x) dx$. How do they compare to the actual value? Why?
HINT: The answer is independent of the equation of the line. What is $f''(x)$?

4. (20 points) Determine whether the following definite integrals converge or diverge. If they converge find their value, if not show why.

a. (10 pts)

$$\int_{-1}^1 \frac{1}{x^2} dx.$$

b. (10 pts)

$$\int_5^{\infty} e^{-3x} dx.$$

5. (20 points)

a. (10 pts) Find the volume obtained when rotating the region bounded by the y -axis, the line $y = 4$ and the function $f(x) = x^2$ about the x -axis.

b. (10 pts) Find the volume of the same region rotated about the y -axis.