Final Exam

Math 10B **Name:** _____
12/08/08 **Section:** _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Give your answers in exact form, for instance:

$$\frac{1}{2}\left(e^{\pi}+e^{2\pi}\right).$$

  as opposed to 279.31617.
- A single 8 1/2 × 11 sheet of notes (double sided) is allowed. Calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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1. (*40 points*)

a. (*10 pts*) The velocity of a car is given by $v(t) = t^2 + 3t - 2$. Find midpoint and trapezoid approximations with 3 boxes of the distance traveled between $t = 1$ and $t = 4$.

b. (*10 pts*) How many boxes must be used so that the left hand approximation of $\int_0^1 e^{-x} dx$ is guaranteed accurate with error at most 0.01?

c. (*10 pts*) Sketch a function $f(x)$ so that

$$LHS(k) \leq TRAP(k) \leq \int_0^1 f(x)dx \leq MP(k) \leq RHS(k).$$

(Where $LHS, TRAP, etc.$ are the left hand sum, trapezoid rule, etc, approximations of $\int_0^1 f(x)dx$. Note you need not give an explicit function, a sketch suffices.

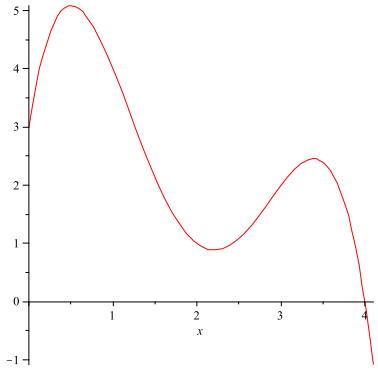
d. (*10 pts*) Give an example of a function $f(x)$ such that $LHS(k) = RHS(k) \neq \int_a^b f(x)dx$ if such exists. If not, explain why.
HINT: Such a function exists.
HINT: Convenient functions exist where $a = 0, b = \pi$ and $a = -1$ and $b = -1$ (though certainly others exist and are fine answers).

2. (*20 points*)

a. (*10 pts*) Find

$$\frac{d}{dx}\int_x^{x^2} e^{2t}dt.$$

b. (*10 pts*) Below is a graph of $f'(x)$ for some function f. NOTE: This is a graph of the *derivative* of f.



Where does f obtain it's maximum on the interval $[0, 4]$?

3. (*20 points*)

a. (*10 pts*) Without calculating the integral, is

$$\int_{-\pi/4}^{\pi/4} \ln(\cos(x))dx.$$

positive or negative. Why?

b. (*10 pts*) Using comparison tests, determine whether

$$\int_{10}^{\infty} \frac{x^2+3x}{x^4-x}dx$$

converges or diverges.

4. (*40 points*) Find the following definite or indefinite integrals:

a. (*10 pts*)

$$\int (\sin(x) + \cos(x))^2 dx.$$

b. (*10 pts*)

$$\int_0^{\pi/2} \sin(x)\cos(x)e^{\sin(x)} dx.$$

c. (*10 pts*)

$$\int_{\sqrt{2}/2}^{1} \frac{1}{(1-x^2)^{3/2}} dx.$$

d. (*10 pts*) Using the book formula

$$\frac{1}{\cos^m(x)} dx = \frac{1}{m-1} \frac{\sin(x)}{\cos^{m-1}(x)} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2}(x)} dx, m \neq 1, \text{ } m \text{ positive.}$$

Find

$$\int \sec^4(3x) dx.$$

5. (*20 points*)

a. (*10 pts*) A building is constructed in the region bounded by the functions $f(x) = 2x$ and $g(x) = x^2$ whose cross-sections cut perpendicular to the x-axis are equilateral triangles. Find the volume of this building.

b. (*10 pts*) .

Find the volume of the region bounded by $f(x) = 1 - x^3$, the x-axis and the y-axis rotated around the line $y = -1$.

6. (*20 points*)

a. (*10 pts*) For what values of α (if any) is $y = e^{\alpha x}$ a solution to the differential equation

$$ay'' + by' + cy = 0.$$

Note: α will be a function of a, b and c.

b. (*10 pts*) Show that $y = A\sin(3x) + B\cos(3x)$ is a solution to the differential equation

$$y'' + y = 0.$$

7. (*20 points*)

a. (*10 pts*) Identify the equilibrium solutions to the differential equation

$$\frac{dy}{dx} = -0.5y(y-4)(y+2)$$

and label them stable or unstable.

b. (*10 pts*) Give an example of a differential equation that has a stable equilibrium at $y = 1$, and an unstable equilibrium at $y = -2$.

8. (*20 points*) A drug is injected into a patient via an IV at constant rate $1mg/hr$. Meanwhile, the drug leaves the body at a rate r proportional to the amount currently in the body. At time $t = 0hr$, no drug is in the patient. At time $t = 1$, there are $.8mg$ of the drug in the patient.
a. (*10 pts*) Set up the differential equation describing this situation. Identify the initial conditions.

b. (*10 pts*) Solve the differential equation, and find an expression for r the rate at which the drug leaves the body. NOTE: You may not be able to actually find r.

9. (*20 points*) **a.** (*10 pts*) Solve the differential equation:

$$\frac{dP}{dt} = \cos(t)\sin^2(P).$$

b. (*10 pts*) Determine (2pts each) whether or not the following are separable differential equations:

$$(a) \quad x + \frac{dy}{dx} = y$$

$$(b) \quad \frac{dP}{dt} = t(P + x)$$

$$(c) \quad \frac{dy}{dt} = e^{y+2t}$$

$$(d) \quad \frac{dy}{dx} = \ln(x + y)$$

$$(e) \quad \frac{dC}{dt} = \ln(C^t)$$