Sample Midterm Exam

Math 20F
8/22/08

Name: _____
Section: _____

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the poin.
- This test has xxx problems and is worth xxx points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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1. (*0 points*) (**a**) Let $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \\ 1 & 2 & 2 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Solve $A\mathbf{x} = b$.

(**b**) Suppose $T\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the matrix for the linear transformation T.

(**c**) Define linear independence.

(**d**) Let $A = \begin{pmatrix} 1 & 2 & -2 \\ 3 & 4 & 0 \\ 2 & 5 & -7 \end{pmatrix}$. Are the columns of A linearly independent?

2. (*0 points*)

(**a**) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. Let $B = \begin{pmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & -1 \end{pmatrix}$. Compute A^T, AB and $B^T - 3A$.

(**b**) Let $A = \begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & 0 \end{pmatrix}$. Compute A^{-1}.

(**c**) Suppose $A \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and $A \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$. Find A^{-1}.

3. (*0 points*) (**a**) Suppose $A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & -3 & 0 & 1 \\ 0 & 1 & 2 & 2 & -1 \\ 3 & 0 & 0 & 0 & 2 \end{pmatrix}$. Compute $\det(A)$.

(**b**) Suppose $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Use Cramer's rule to solve $A\mathbf{x} = \mathbf{b}$.

(**c**) Find the area of the *triangle* with vertices $(2, 3)$, $(4, 7)$ and $(8, 4)$ using the methods of this course.

4. (*0 points*) For each statement, mark it true of false. If it is false give a (counter)example or brief proof. If it is true give a reason - if the reason is a theorem, state the theorem, otherwise give a brief proof. No credit for answers without a correct reason or example. Unless explicitly noted, there are no condition on the dimensions of matrices A and B.

(**a**) If $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ are linearly dependent, then one of the vectors is a multiple of another.

(**b**) If $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b}$, the columns of A span $\mathbf{R}^m$.

(**c**) If $AB = AC$, then $B = C$.

(**d**) If the transformation $T(\mathbf{x}) = AB\mathbf{x}$ is onto, then the transformation $T'(x) = A\mathbf{x}$ is onto

(**e**) If A and B are $n \times n$, and invertible, then $A + B$ is invertible.

(**f**) If A is row equivalent to B, then $\det(A) = \det(B)$.

(**g**) If the columns of A are linearly dependent, then $\text{Col}(A)$ is not a vector space.

5. (*0 points*)

(**a**) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation verify that the range of $T = \{\mathbf{v} \in \mathbb{R}^m : T(\mathbf{x}) = \mathbf{v} \text{ for some } \mathbf{x} \in \mathbb{R}^n\}$, is a vector space. (Note the range is not necessarily all of $\mathbb{R}^m$. $\mathbb{R}^m$ is the co-domain, not the range.).

(**b**) Verify that the derivative operator $\frac{d}{dx}$ is a linear transformation from the vector space of polynomials of degree at most 3 to the vector space of polynomials of degree at most 2. Is this linear transformation $1-1$? Onto?

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